

GRADE 3 MATH: COOKIE DOUGH

UNIT OVERVIEW

This packet contains a curriculum-embedded CCLS aligned task and instructional supports. The task is embedded in a 4-5 week unit on interpreting and linking representations, modeling situations, solving non-routine problems and justifying arguments of multiplication and division.

TASK DETAILS

Task Name: Cookie Dough

Grade: 3

Subject: Mathematics

Task Description: The tasks in the unit access the full range of *Depth of Knowledge*, including Recalling and Recognizing, Using Procedures, Explaining, Concluding and Making Connections, Extensions and Justifying.

Standards:

- 3.OA.1** Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each.
- 3.OA.2** Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.
- 3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- 3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers.
- 3.OA.5** Apply properties or operations as strategies to multiply and divide.
- 3.OA.6** Understand division as an unknown-factor problem.
- 3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
- 3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- 3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

Standards for Mathematical Practice:

- MP.1** Make sense of problems and persevere in solving them.
- MP.2** Reason abstractly and quantitatively.
- MP.3** Construct viable arguments and critique the reasoning of others.
- MP.4** Model using mathematics.



TABLE OF CONTENTS

The task and instructional supports in the following pages are designed to help educators understand and implement tasks that are embedded in Common Core-aligned curricula. While the focus for the 2011-2012 Instructional Expectations is on engaging students in Common Core-aligned culminating tasks, it is imperative that the tasks are embedded in units of study that are also aligned to the new standards. Rather than asking teachers to introduce a task into the semester without context, this work is intended to encourage analysis of student and teacher work to understand what alignment looks like. We have learned through the 2010-2011 Common Core pilots that beginning with rigorous assessments drive significant shifts in curriculum and pedagogy. Universal Design for Learning (UDL) support is included to ensure multiple entry points for all learners, including students with disabilities and English language learners.

PERFORMANCE TASK: COOKIE DOUGH.....	3
RUBRIC.....	6
SCORING GUIDE.....	7
PERFORMANCE LEVEL DESCRIPTIONS.....	8
ANNOTATED STUDENT WORK.....	9
INSTRUCTIONAL SUPPORTS.....	19
UNIT OUTLINE.....	20
INITIAL ASSESSMENT: SPONSORED WALK.....	27
FORMATIVE ASSESSMENT:	
INTERPRETING MULTIPLICATION & DIVISION.....	30
FORMATIVE ASSESSMENT: SQUIRRELING IT AWAY.....	57
SUPPORTS FOR ENGLISH LANGUAGE LEARNERS	62
SUPPORTS FOR STUDENTS WITH DISABILITIES.....	65



GRADE 3 MATH: COOKIE DOUGH PERFORMANCE TASK

Cookie Dough

Clear Creek School is fundraising.
They are selling Cookie Dough in tubs.



Chocolate Chip Cookie Dough \$5 a tub	Peanut Butter Cookie Dough \$4 a tub	Oatmeal Cookie Dough \$3 a tub
--	---	---

1. Jill sold 2 tubs of Oatmeal Cookie Dough.

How much did she raise? \$ _____

2. Joe sold 4 tubs of Peanut Butter Cookie Dough and 4 tubs of Chocolate Chip Cookie Dough.

How much money did he raise in all? \$ _____

Show how you figured it out.

3. Jade sold only Peanut Butter Cookie Dough. She raised \$32.

How many tubs did she sell? _____ tubs

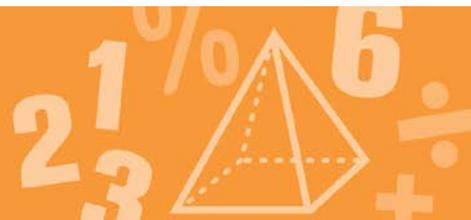
Show how you figured it out.

4. Jermaine's mother loves oatmeal cookies. She has \$20. to spend.

What is the greatest number of tubs of Oatmeal Cookie Dough she can buy?

_____ tubs

Explain how you figured it out.



GRADE 3 MATH: COOKIE DOUGH

RUBRIC

The rubric section contains a scoring guide and performance level descriptions for the Cookie Dough task.

Scoring Guide: The scoring guide is designed specifically to each small performance task. The points highlight each specific piece of student thinking and explanation required of the task and help teachers see common misconceptions (which errors or incorrect explanations) keep happening across several papers. The scoring guide can then be used to refer back to the performance level descriptions.

Performance Level Descriptions: Performance level descriptions help teachers think about the overall qualities of work for each task by providing information about the expected level of performance for students. Performance level descriptions provide score ranges for each level, which are assessed using the scoring guide.

Grade 3 Math: Cookie Dough

Cookie Dough Scoring Guide

Cookie Dough		Rubric	
<p>The core elements of performance required by this task are:</p> <ul style="list-style-type: none"> work with multiplication and division in a real context <p>Based on these, credit for specific aspects of performance should be assigned as follows:</p>		points	section points
1.	Gives correct answer: \$6.00 and shows some correct work such as: $3 + 3 = 6$	1	1
2.	Gives correct answer: \$36.00 Shows work such as: 4×4 and 4×5	1 1	2
3.	Gives correct answer: 8 Shows work such as: $32 \div 4$	1 1	2
4.	Gives correct answer: 6 Gives a correct explanation such as: The most her mother can buy is 6 tubs because I counted by 3s and she can buy 6 but doesn't have enough for 7 or, I counted 3, 6, 9, 12, 15, 18, 21. She doesn't have 21.	1 1	2
Total Points			7

Grade 3 Math: Cookie Dough Rubric

Performance Level Description and Cut Scores

Performance is reported at four levels: 1 through 4, with 4 as the highest.

Level 1: Demonstrates Minimal Success (0 – 1 point)

The student's response shows few of the elements of performance that the tasks demand as defined by the CCSS. The work shows a minimal attempt on the problem and struggles to make a coherent attack on the problem. Communication is limited and shows minimal reasoning. The student's response rarely uses definitions in his/her explanations. The student struggles to recognize patterns or the structure of the problem situation.

Level 2: Performance Below Standard (2 – 3 points)

The student's response shows some of the elements of performance that the tasks demand, and some signs of a coherent attack on the core of some of the problems, as defined by the CCSS. However, shortcomings are substantial, and evidence suggests the student would not be able to produce high-quality solutions without significant further instruction. The student might ignore or fail to address some of the constraints of the problem. The student may occasionally make sense of quantities in relationships within the problem, but their use of quantity is limited or not fully developed. The student response may not state assumptions, definitions, and previously established results. While the student makes an attack on the problem, it is incomplete. The student may recognize some patterns or structures, but has trouble generalizing or using them to solve the problem.

Level 3: Performance at Standard (4 – 5 points)

For most of the task, the student response shows the main elements of performance the tasks demand as defined by the CCSS. The response is organized as a coherent attack on the core of the problem. There are errors or omissions, some of which may be important, but of a kind that the student could well fix, with more time for checking, revision and some limited help. The student explains the problem and identifies constraints, they make sense of quantities and their relationship to the problem situations. S/he often uses abstractions to represent a problem symbolically or with other mathematical representations. The student response may use assumptions, definitions, and previously established results in constructing arguments. S/he may make conjectures and build a logical progression of statements to explore the truth of their conjectures. The student might discern patterns or structures and make connections between representations.

Level 4: Achieves Standards at a High Level (6 – 7 points)

The student's response meets the demands of nearly all of the tasks as defined by the CCSS, with few errors. With some more time for checking and revision, excellent solutions would seem likely. The student response shows understanding and use of stated assumptions, definitions and previously established results in the construction of arguments. The student is able to make conjectures and build a logical progression of statements to explore the truth of his/her own conjecture. The student response routinely interprets his/her mathematical results in the context of the situation and reflects on whether the results make sense. The communication is precise, using definitions clearly. The student looks closely to discern a pattern or structure. The body of the work looks at the overall situation of the problem and process, while attending to the details.



GRADE 3 MATH: COOKIE DOUGH

INSTRUCTIONAL SUPPORTS

The instructional supports on the following pages include a unit outline with formative assessments and suggested learning activities. Teachers may use this unit outline as it is described, integrate parts of it into a currently existing curriculum unit, or use it as a model or checklist for a currently existing unit on a different topic.

UNIT OUTLINE.....	20
INITIAL ASSESSMENT: SPONSORED WALK.....	27
FORMATIVE ASSESSMENT:	
INTERPRETING MULTIPLICATION & DIVISION.....	30
FORMATIVE ASSESSMENT: SQUIRRELING IT AWAY.....	57

Unit Outline Template

INTRODUCTION: This unit outline provides an example of how teachers may integrate performance tasks into a unit. *Teachers may (a) use this unit outline as it is described below; (b) integrate parts of it into a currently existing curriculum unit; or (c) use it as a model or checklist for a currently existing unit on a different topic.*

Grade 3: Interpreting Multiplication and Division

UNIT TOPIC AND LENGTH:

- This unit should run between 20 and 25 standard periods of instruction. One of the periods will involve the pre-assessment (0.5 period), introducing and supporting problem solving on the long lesson (2 periods), teaching the formative assessment lesson (2.5 periods) and the final assessment (0.5 period). This unit should be taught after or during the time students have learned about multiplication and division.

COMMON CORE LEARNING STANDARDS:

- **3.OA.1** Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each.
- **3.OA.2** Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.
- **3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- **3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers.
- **3.OA.5** Apply properties or operations as strategies to multiply and divide.
- **3.OA.6** Understand division as an unknown-factor problem.
- **3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
- **3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- **3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.
- **MP.1** Make sense of problems and persevere in solving them.
- **MP.2** Reason abstractly and quantitatively.
- **MP.3** Construct viable arguments and critique the reasoning of others.
- **MP.4** Model using mathematics.

<p>BIG IDEAS/ENDURING UNDERSTANDINGS:</p> <p>Student will understand:</p> <ul style="list-style-type: none"> ➤ How to match equations to language descriptions. ➤ How to describe multiplicative and division relationships with area models and discrete models. ➤ How to connect naked number equations to contextual word problems. ➤ How to convince others that different multiplication and division representations can be equivalent. 	<p>ESSENTIAL QUESTIONS:</p> <ul style="list-style-type: none"> ➤ What are the various representations of multiplication and division (<i>finding an unknown product, and finding an unknown factor</i>)?
<p>CONTENT:</p> <ul style="list-style-type: none"> ➤ The big idea of the unit is to understand the meaning of multiplication and its inverse relationship to division. The understanding should include the ability to translate between different representations of multiplication and division including the language of mathematics: understanding equations written in terms of equal-sized groups, area models, discrete models, and using contextual word situations to model multiplication and division. ➤ Students use understanding of equivalency to match multiplication and division equations to various visual representations and verbal descriptions. ➤ Students demonstrate knowledge through the expert investigation, the performance assessment task in the formative assessment lesson and the final assessment. 	<p>SKILLS:</p> <ul style="list-style-type: none"> ➤ The targeted proficiencies; technical actions and strategies. Starting with an action verb. ➤ Students match multiplication and division equations with the language of mathematics, area models, discrete models, and contextual word problems. ➤ Students apply what they know about the meaning of multiplication and division to generate visual, verbal and numerical representations. ➤ Students look at problem situations and translate them into numerical and visual representations. ➤ Students apply the new knowledge to constructing arguments, making connections between representations and carefully evaluate the arguments and justifications of others.

ASSESSMENT EVIDENCE AND ACTIVITIES:

INITIAL ASSESSMENT :

The unit begins with the performance task *Sponsored Walk*. The task is designed to measure what students bring to the unit with regard to their knowledge and skill at working with multiplication and division. Please reference *Sponsored Walk* for full details.

FORMATIVE ASSESSMENT:

About 3/4 of the way through the unit, teachers would use the *formative assessment lesson(FAL)*. The FAL is entitled *Interpreting Multiplication and Division*. A different pre-assessment task should be administered in class at least two days prior to the two-day lesson. Students should spend no more than 20 minutes on the task. Teachers should review the student work prior to teaching the lesson. The FAL comes with complete teacher notes and student pages. Please reference *Interpreting Multiplication and Division* for full details.

FINAL PERFORMANCE TASK:

The final performance assessment is entitled *Cookie Dough*. It should be administered during a class period. Most students will complete the task in about 10 – 20 minutes, although time should not be a factor. The teacher should provide a reasonable amount of time for all students to finish. The students should be allowed to use any tools or materials they normally use in their classroom. The task can be read to the students and all accommodations delineated in an IEP should be followed. Please reference *Cookie Dough* for full details.

LEARNING PLAN & ACTIVITIES:

- The unit is designed with a pre-assessment task, an expert task/investigation, a formative assessment lesson and a final assessment. The mathematics of the unit involves understanding the meaning of multiplication and division, as well as interpreting and translating between representations. Students will work with multiplication and division using equations, mathematical language, area models, discrete representations, number lines, and contextualized word problems to think about the meaning of multiplication and its inverse, division. Also included are teacher notes, rubrics, and sample student work with commentary. This unit is designed to accompany the curriculum a teacher currently uses to teach the topics listed. The elements in the unit will provide activities to foster formative assessment practices, conceptual understanding and non-routine problem solving.
- The expert investigation is entitled *Squirreling it Away*. It contains three separate but mathematically related problems labeled Part A, Part B, and Part C. All students should start with the Part A task and then proceed at their own speed to Part B and Part C. It is more important for the student to work deeply on a part and complete a write up, than to

merely work through and find answers. It is the student's responsibility to be reflective and thorough in their explanations, findings and justifications.

- **Re-engagement:** The unit begins with a pre-assessment called sponsored walk. After the teacher analyzes the student performances, a decision of how to proceed through the lesson is important. Instead of going back to re-teach skill and concepts lacking, it is much more powerful for the teacher to use the work students have already done on a contextual problem, to help them build upon their understanding from previous thinking. That process is called "re-engagement". It is powerful to use student work because students become very engaged in the process of figuring out what someone else is thinking. This process of analyzing and contrasting student thinking raises the cognitive demand for students and supports them to be more reflective about their own thinking. The re-engagement lesson will depend upon the results from the students in each individual class. Thus, each lesson will look very different from class to class. Students have already done the task on their own and now the important ideas need to be brought out and examined. In the process, students must have the opportunity to confront and understand the error in the logic of their misconceptions. Often, as teachers, we try to prevent errors by giving frequent reminders, such as "line up the decimal point," but actually errors provide great learning opportunities for all students. Students don't let go of misconceptions until they understand why they don't make sense. For the student, there is underlying logic to their misconceptions.

Re-engagement – Confronting misconceptions, providing feedback on thinking, going deeper into the mathematics.

1. Start with a foundational problem to bring all the students along; this allows students to clarify and articulate important mathematics in order to better understand the entirety of the task.
2. Share different student approaches and ask all students to make sense of each strategy. Have all students compare the strategies to look for the mathematical connections and relationships.
3. Have students analyze misconceptions and discuss why they don't make sense. In the process students can let go of misconceptions and clarify their thinking about big mathematical ideas.
4. Have students determine how a strategy could be modified to get the correct solution. Have students look for the seeds of mathematical thinking in the selected student work.

- **Number Talks** A daily ritual with the entire class for the purposes of developing conceptual understanding of numbers, operations and mathematics. Number talks are used to:
 - Review and practice operations, procedures and concepts of numbers.
 - Introduce concepts and properties about numbers.
 - Reinforce procedures and number concepts.
 - Explore connections about numbers.

Do a number talk every day but for only 10 minutes. A few minutes more often is better than a lot of minutes infrequently.

1. Ask questions such as
 - How did you think about that?
 - How did you figure it out?
 - What did you do next?
 - Why did you do that? Tell me more.

- Who would like to share their thinking?
 - Did someone solve it a different way?
 - Who else used this strategy to solve the problem?
 - What strategies do you see being used?
 - Which strategies seem to be efficient, quick, and simple?
2. Give yourself time to learn how to
 - Record student solutions
 - Listen to and observe students
 - Collect notes about student strategies and understanding
 3. Name/label the strategies that emerge from your students:
 - Use doubles
 - Break apart numbers
 - Make it simpler
 - Use landmark numbers (25, 50, 75, 200, etc)
 - Use a model to help
 - Use what you already know
 - Make a “10”
 - Start with the 10’s
 - Think about multiples
 - Think about money
 - Traditional algorithm
 - Counting on
 4. Create a safe environment. When students feel safe, they are comfortable sharing answers even when it’s different from everyone else’s.
 5. Give opportunities for students to “think first.”
 6. Encourage self-correction; it’s okay to change your mind, analyze your mistake, and try again.
 7. Give number talks time to become part of your classroom culture. Expect them to follow the usual learning curve stages. “Keep on keeping on” and you will get positive results.
- **Think/Write/Pair/Share** is a high leverage strategy that respects individual time to process and organize ideas before engaging in peer-to-peer discussions. This process can be used throughout the unit as a vehicle for students to self reflect, construct new meaning by building on the ideas of others, and strengthen their arguments.
 - **Journal Entries for Reflection:** Using a prompt such as, “*How has my thinking changed as a result of what I have discussed with my peers?*” or “*How can I improve my argument or explanation using evidence and content vocabulary?*” can provide valuable opportunities for students to tweak their own solutions, during class or for homework, and subsequently, deepen their understanding of content.
 - **Purposeful Questioning and Feedback** are instructional supports that can help refocus students’ attention to specific aspects of their work. There are some suggestions based on some common difficulties. Although these error patterns/questions relate to the pre-task *Sponsored Walk*, they can be easily modified to address similar misconceptions that are revealed from any other problems or tasks used:

Points	Understandings	Misunderstandings	Suggested Prompts and Questions
0	95% of the students with this score attempted the task.	Students had difficulty choosing the correct operation needed to solve the problem. Many students chose addition for all problems or multiplication for all problems. <i>Example: 6 laps x \$30 total = \$180 per lap</i>	<ul style="list-style-type: none"> • Why did you pick that operation for the problem? • How did you know when to add and when to multiply?
2	Students with this score generally knew to add the amount of money per lap for Jack and Bill, and could total that to \$10.	The students did not know to take this rate and multiply it by the number of laps to find the amount of money earned.	<ul style="list-style-type: none"> • How do you find how much money was earned? • How much would be earned if they did just two laps?
4	Students could use all the facts in part one to find the total amount of money earned by Jack and Bill and show all their steps.	More than 15% of the students had a final answer of \$10 for part 1. 6% of the students added all the numbers in part 1 to get an answer of 15. Adding dollars and laps together didn't bother students.	<ul style="list-style-type: none"> • Does it make sense to add dollars and laps together? • What would you label your answer if you did?
6	Students could find the total money earned by Jack and Bill. They could also work backwards from the total amount of money to find the number of laps Maria ran.	8% of the students multiplied instead of divided in part 2. 13% of the students could write a correct number sentence for part 2, but could not decipher which part of the number sentence represented the answer (picking either the 30 or 6 instead of the 5). 7% of the students just added the numbers in the problem, not recognizing the operation appropriate for solving the question.	<ul style="list-style-type: none"> • Why did you multiply those values, what did you think you would find out? • What might a number sentence look like for the total money being earned by Jack and Bill? • How might you check your answer to see if it is right?
7	Students could understand the context of part 3, but weren't successful with interpreting the constraint "at least" or understanding how to interpret a remainder. Many students used multiplication to think about the problem and arrive at a correct or nearly correct solution.	Students were thinking about $6 \times 3 = 18$ and $7 \times 3 = 21$. They picked 6 laps because they didn't want to go over \$20. They did not understand how the remainder in 20 divided by 3 applied to the context of earning a given amount of money.	<ul style="list-style-type: none"> • What does the remainder 2 mean in this problem? • What does "at least" mean for this problem? • If Sarah does 6 laps how much money will she raise? Is that enough? • Is this easier to think about using multiplication or division? Why?

RESOURCES:

- Normal materials used in math class include manipulatives such as cards for matching activities, square tiles, counters, and cm graph paper.
- All the materials referenced in the assessments, formative assessment lesson, and expert investigation are included. Most supplementary materials are located in the appendix, including the established scored benchmark papers and some student work examples.
- What isn't included in print materials can be found on a TBA websites.



GRADE 3 MATH: COOKIE DOUGH

INITIAL TASK: SPONSORED WALK

Sponsored Walk

This problem gives you the chance to:

- choose and use number operations in a real context
-



Students at the Mountain View Elementary School do a sponsored walk.

1. Jack is sponsored for \$6 for each lap.

Bill is sponsored for \$4 for each lap.

Jack and Bill each do 5 laps.

How much money do Jack and Bill raise in all? \$ _____

Show your work.

2. Maria does 6 laps.

She raises \$30.

How much for each lap was she sponsored? \$ _____

Show how you figured it out.

3. Sarah wants to raise at least \$20.

She is sponsored for \$3 for each lap.

What is the least number of whole laps she must walk? _____

Explain how you figured it out.



GRADE 3 MATH: COOKIE DOUGH

FORMATIVE ASSESSMENT LESSON: INTERPRETING MULTIPLICATION AND DIVISION

Interpreting Multiplication and Division

Silicon Valley Mathematics Initiative- 2011

Formative Assessment Lesson

Interpreting Multiplication and Division

Mathematical Goals

This lesson unit is designed to assess students' understanding of multiplication and division and the relationship between these operations, as well as students' ability to translate between different representations or models of these operations. The representations include symbolic, concrete, bar and area, number line, and contextual models of these number operations. The lesson design facilitates identification of students who have difficulty:

- recognizing multiplicative relationships and transitioning from additive to multiplicative solution strategies
- understanding and using the language of “equal groups of” and “equal parts of” to make sense of multiplication and division
- translating between models
- understanding the meanings of the words factor, multiple, product, and quotient

Standards Addressed:

- Third Grade Operations and Algebraic Thinking- Represent and solve problems involving multiplication and division
- Third Grade Operations and Algebraic Thinking– Understand properties of multiplication and the relationship between multiplication and division
- Third Grade Operations and Algebraic Thinking – Multiply and divide within 100
- Third Grade Operations and Algebraic Thinking – Solve problems involving the four operations and identify and explain patterns in arithmetic

This lesson also relates to the following Mathematical Practices in the Common Core State Standards for Mathematics:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

Students will be asked to do a written reflection on how they were demonstrating these Practice Standards throughout their work in this unit.

Essential Understandings:

The big idea of the unit is to understand the meaning of multiplication and its inverse relationship to division. The understanding should include the ability to translate between different representations of multiplication and division. This lesson is designed to facilitate the following understandings:

- Student will understand:
 - How to match equations to language descriptions
 - How to describe multiplicative and division relationships with area models and discrete models
 - How to connect naked number equations to contextual word problems

- How to convince others that different multiplication and division representations can be equivalent

Introduction:

This lesson unit is structured in the following way:

- Students work on their own, completing an assessment task that is designed to reveal their current understandings and misconceptions.
- Students work in pairs or threes on collaborative discussion tasks. As they work, they translate between the symbols and models of multiplication and division and build understanding of the relationships between these operations through discussion as they make sense of the models.
- Students return to their original work and try to improve their own responses.

Materials required:

Each student will need one copy of the assessment task:

Each pair will need the following: (the cards should be cut up and packaged beforehand).

- Card Set A Equations; Card Set B Verbal (Explanations in Words); Card Set C Area Models; Card Set D Concrete Models; Card Set E Contextual Models; Card Set F Number Line Models
- Pattern blocks, Cuisenaire rods, counters, grid paper, and blank paper should be made accessible to students
- A glue stick, a felt tipped pen, a large sheet of paper or card for making a poster-one per pair
- Envelopes and paper clips for storing matched cards in between lessons if time is short
- Additional supplies dependent on procedures used for sharing and debriefing student work, such as overhead transparencies, document reader, additional chart paper

Time needed:

The lesson will need at least two one-hour sessions. Timings given are only approximate. Exact timings will depend on the needs of the class.

Before the lesson:

Individual Assessment Task: “Baking Cookies”

The assessment task, “Baking Cookies” should be completed before the lesson. Ask students to attempt the task on their own. Explain that they should not worry too much if they cannot understand or do everything, because you plan to teach a lesson using a similar task which should help them.

It is important that students are allowed to answer the questions without assistance, as far as possible. If students are struggling to get started, ask questions that help them understand what is required, but don't do it for them!

Assessing student responses:

Collect a sample of students' responses to the task and make some notes on what their work reveals about their current levels of understanding. The purpose of doing this is to forewarn you of the difficulties students may experience during the lesson itself, and so that you may prepare carefully. Do not grade students' work at this stage. Research shows this could be counterproductive as it will encourage students to compare their grades, and will distract their attention from the mathematics. Instead, try to understand their reasoning and think of ways in which you can help them.

Suggested Lesson Outline

Class introduction- Math Talk (15 minutes)

[Please see Math Talk Format below for support]

MATH TALK FORMAT: IMPLEMENTING NUMBER TALKS

Helpful Hints

- Do number talks every day, but for only 10 minutes. A few minutes more often is better than a lot of minutes infrequently.
- Ask questions such as....
 - How did you think about that?
 - How did you figure it out?
 - What did you do next?
 - Why did you do that? Tell me more.
 - Who would like to share their thinking?
 - Did someone solve it a different way?
 - Who else started the problem this way?
 - Who else used this strategy to solve the problem?
 - What strategies do you see being used?
 - Which strategies seem to be efficient, quick, simple?
- Experiment with using the overhead, the whiteboard, chart paper, etc.
- Consider having students "circle up" in chairs or on the floor.
- Give yourself time to learn how to....
 - record student solutions
 - listen to and observe students
 - collect notes about student strategies and understanding
- To help determine what numbers or problems you select use what you learn from previous number talks as well as the focus of your daily classroom instruction.
- Do number talks with yourself and others to try new strategies and increase your own confidence.
- Name/label the strategies that emerge from your students:

• Use doubles	• Break apart numbers	• Make it simpler
• Use landmark numbers (25, 50, 75, 200 etc.)	• Use a model to help	• Use what you already know

<ul style="list-style-type: none"> • Make a "10" 	<ul style="list-style-type: none"> • Start with the 10's 	<ul style="list-style-type: none"> • Think about multiples
<ul style="list-style-type: none"> • Think about money 	<ul style="list-style-type: none"> • Traditional algorithm 	<ul style="list-style-type: none"> • Counting on

- Use related problems: 3×14 , 3×114 , 3×1014 or $7 + 8$, $27 + 8$, $107 + 8$, or 3×7 , 6×7
- Do number talks in small groups
- Ask students to: "Do as much of the problem as you can."
- Give students lots of practice with the same kinds of problems.
- Use numbers for subtraction and addition that require students to work past a ten or hundred.
Example: $56 + 7 = 87 - 9 = 25 + 6 = 94 + 8 = 106 - 8 =$
- Give students opportunities to add and subtract 9, then 8 etc. using 10 as a friendly number to work with.
Example: $68 + 10 = 78$ so $68 + 9 = 77$
- Expect students to break apart numbers, not count on their fingers. Show them how.
 $6 + 8$ (think of 6 as $4 + 2$; add the 2 to 8 to get 10 and just add the remaining 4 to get 14)
- Show the strategy you used. Make sure they know it's not "the" way, just another strategy.
- Give students larger numbers so they can give "estimates."
- If you use chart paper, write down the student's name next to their solution. Keep track of who is participating and their strategies. Use the following as a "sorting" or assessment guide:
 - Can they figure it out (by counting on, using an involved strategy, etc.)
 - Begin to use efficient strategies (can they complete some of the problem efficiently)
 - Do they just know or are they using efficient strategies
- Create a safe environment. When children feel safe, they are comfortable sharing an answer even when it's different from everyone else's.
- Provide concrete models (snap cube "trains", base 10 blocks, money etc.)
- Give opportunities for children to "think first" and then check with the models.
- Have students occasionally record their thinking and the steps they use to solve a problem.
- Encourage self-correction; it's okay to change your mind, analyze your mistake, and try again.
- Provide number stories.
- Be curious; avoid making assumptions.
- Give number talks time to become part of your classroom culture. Expect them to follow the usual learning curve stages. "Keep on keeping on" and you will get positive results!

KIDS LIKE NUMBER TALKS!

Present the following multiplication problem to students using a Math Talk Format. Tell them that you are not interested in the answer as much as how they figure it out.

Mrs. Clark's class has 36 students. She wants to put their desks into 6 rows. How many student desks will she need to put in each row?

First, ask students to use a manipulative to demonstrate their thinking at their desks. These could be color tiles or cm cubes. Have students share with their shoulder partner and have a conversation about the similarities or differences between the two representations.

Ask for student volunteers to share one of their solution strategies. Have each volunteer, use his/her white board to explain one strategy/model. Allow students to ask questions of the student presenting to ensure clarification of the solution strategies. If students show any incorrect answers, write the correct answer on the board and discuss the reasoning as a group.

Second, ask students to write an equation that could be used to solve the answer on a white board or facsimile thereof, [teachers have put tag board inside of a plastic sleeve and given students overhead transparency pens or washable markers to use to write on these.]

On your mini-white boards, show me.....

Put their ideas on the classroom whiteboard/document reader/smart board and hold a discussion about which one[s] represent the mathematics of the problem and could be used to find the answer.

Show students an area model representation of this problem and ask students if this model represents the given multiplication word problem. Give them a few minutes of think time, and then ask them to discuss with their elbow partner. Have a whole group discussion to surface understandings and misconceptions.

If needed, show another area model and ask students to write an equation to represent this new area model. Likewise, having students write a contextual problem with a partner would connect these representations together.

LESSON: To deepen student understanding of multiplication and division, use the terms *factor, multiple, product and quotient* during discussions of the different representations i.e., symbols, words, and concrete/bar/area/number line models.

Collaborative Task # 1: Matching symbolic notation of equations (Card Set A) with verbal explanations (Card Set B) (15-20 minutes)

The first task is designed to help students identify and interpret equations involving multiplication and division and to match the equations with verbal descriptions [explanations in words], using such language as “groups of” and “as big as.” Organize students into pairs and give out Card Set A – Equations, and Card Set B – Verbal Explanations.

*Your task is to match the equations (symbolic notations) **Card Set A** with the explanations in words **Card Set B**.*

Take turns with your partner choosing an equation, and the verbal description to match it.

Place these side by side on the table and explain to your partner[s] how they match.

If you cannot find a matching card, you have to write your own using one of the blank cards.

Take turns choosing an equation and the verbal description to match it with your partner. Place these side by side on the table and explain to your partner[s] how they match.

It is extremely important to model this behavior with students in the class. This may be a student's first experience having a structured mathematical dialogue with another student.

There is a one to one correspondence between sets. The blank cards are an integral part of the sets because some students may not see the corresponding verbal description and will choose to use the blank cards to complete their match. This provides invaluable information to you, the teacher, about student understanding and misconceptions.

Furthermore, to increase the cognitive demand of these collaborative tasks, selecting a few cards of your choice to be removed from one or more sets of representations, will force students to fill in the missing representation[s] on their own, giving deeper insight into student thinking and understanding.

Whether you use the complete set of cards or you remove some, letting students know that there are blank cards available is important because it will reveal additional information of students' understandings and misconceptions if they use them.

Students may notice that some of the equations are related. You do not need to comment on this now, but this is an important connection that warrants discussion as the students progress through the matches. These relationships will be emphasized during the summary processing discussion.

Collaborative Task # 2: Matching symbolic notations (Card Set A), explanations in words (Card Set B), and area models (Card Set C) for multiplication/division (15-20 minutes)

Give each pair of students a copy of **Card Set C**, a glue stick.

*Your task is to match the equations in **Card Set A** with the verbal explanations in **Card Set B** and select the proper area model in **Card Set C**.*

Take turns with your partner choosing an area model Card C, to match one set of Cards A and B.

Place the chosen C card along side the A and B card set on the table and explain to your partner how they match.

If you cannot find a matching card, write your own using one of the blank cards.

Collaborative Task # 3: Matching symbolic notations (Card Set A), verbal explanations (Card Set B), area models (Card Set C), and concrete models (Card Set D) for multiplication/division (15 to 20 minutes)

*Your task is to match the equations in **Card Set A**, the verbal explanations in **Card Set B**, the area models in **Card Set C** with the concrete models in **Card Set D**.*

Take turns with your partner choosing a concrete model Card D to match one set of Cards A, B, and C.

Place the chosen D card along side the A, B, and C card set on the table and explain to your partner how they match.

If you cannot find a matching card, write your own using one of the blank cards.

Collaborative Task # 4: Matching symbolic notations (Card Set A), **verbal explanations (Card Set B), **area models** (Card Set C), **concrete models** (Card Set D), **and contextual (word problems)** (Card Set E) **for multiplication/division** (15 to 20 minutes)**

*Your task is to match the equations in **Card Set A**, the verbal explanations in **Card Set B**, the area models in **Card Set C**, the concrete models in **Card Set D** with the contextual models in **Card Set E**.*

Each of the Set E cards is a representation of a multiplication or division or story. You are to match these situation cards to those already on the table. Remember, if you cannot find a matching card, write your own using one of the blank cards.

Take turns with your partner choosing a number line Card E to match one set of Cards A, B, C, and D.

Place the chosen E card along side the A, B, C, and D card set on the table and explain to your partner[s] how they match.

If you cannot find a matching card, write your own using one of the blank cards.

Collaborative Task #5: Matching symbolic notations (Card Set A), **verbal explanations (Card Set B), **area models** (Card Set C), **concrete models** (Card Set D), **and contextual models** (Card Set E) **for multiplication/division number line models** (Card Set F), (15 to 20 minutes)**

*Your task is to match the equations in **Card Set A**, the verbal explanations in **Card Set B**, the area models in **Card Set C**, the concrete models in **Card Set D**, the contextual models in **Card Set E**, with the number line models in **Card Set F**.*

Take turns with your partner choosing a contextual model Card F to match one set of Cards A, B, C, D, and E.

Place the chosen F card along side the A, B, C, D, and E card set on the table and explain to your partner[s] how they match.

When your group reaches agreement, paste down your final arrangement onto the large sheet, creating a poster.

These posters will be used for the summary class discussion. As students do the matching and pasting or gluing, go around and encourage students to explain why particular cards go together.

Summary Class Work

First, give the following prompt questions to the class. Each pair of students will prepare their responses before presenting their poster and responses to the whole class. **Collect these for your information.**

Individually, think and write about these questions I am going to give you. Share your thoughts with your partner and agree on one response to present to the class. Remember to use mathematical language in your presentation: factor, multiple, product, quotient, "equal groups of", "as big as", etc.

What determined how you matched the cards?

Which representation[s] made the most sense to you? Why?

Which representation[s] was/were the most difficult for you to understand? Why?

Have each pair of students present their poster and responses to the whole class. If possible, limit presentations to about 2 minutes each. Whether or not clarifying questions from the audience are allowed, is dependent upon the current classroom norms, culture and expectations in individual classrooms.

Second, after the completion of presentations and collection of individual responses, re-visit the Standards for Mathematical Practices listed in the beginning of this unit with your students. Ask them to reflect upon these practices and write about how this unit helped them practice one or more of these standards.

Individual work (15-20 minutes)

Lastly, students need to revisit the original task, *Baking Cookies*. Giving students a new, clean copy of this task allows students the opportunity to share their new learning. A potential follow up activity is to have students look at their first work on the task, and compare and contrast it to their second attempt. This affords them the opportunity to articulate and summarize their mathematical understandings by re-engaging in the mathematics of the task. Another option is to have students edit their original paper with a pen. Editing is an invaluable skill used quite effectively in language arts and very little, if at all, in mathematics. Many students struggle with editing their original task work, yet both teachers and students alike find more evidence of learning when students have the opportunity to re-work a clean copy of the original task. This is not to say that teachers and students should not learn to persevere when it comes to editing mathematics. In fact, teachers should attempt to create more opportunity for students to edit and re-edit their mathematics within their classwork.

Solutions:

This table is for your convenience only. Rather than referring to the cards by letter and number, the connections should be made by the content of the cards as there are different possible groupings of six representations.

Equation	Verbal	Area	Concrete	Number Line	Contextual
A1	W7, W8	AM8	CM4, 6, 11	W1	NL7
A2	W 9	AM9	CM8	W2	NL9
A3	W 6	AM6	CM3,CM10	W3	NL6
A4	W1, W4, W12	AM10	CM1	W4	NL11, NL12
A5	W1	AM1	CM12	W5	NL1
A6	W10	AM11	CM9	W6	NL10
A7	W11	AM5	CM2	W7	NL11
A8	W8	AM12	CM11	W8	NL7, NL8
A9	W5, W6	AM6	CM3, CM10	W3	NL5, NL6
A10	W13	AM3, AM7	CM5, CM7	W9	NL4
A11	W1, W2	AM1, AM4	CM11	W5, W10	NL2
A12	W3, W4	AM3	CM7	W9	NL3 with discussion
A13	W4, W13	AM2	CM7	W12	NL3

Answer Key for #s 1 & 2

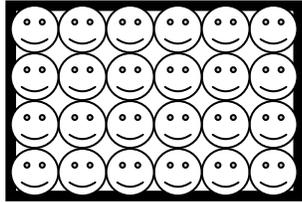
See Rubric for “Baking Cookies

Rubric

Baking Cookies	
<p>The core elements of performance required by this task are:</p> <ul style="list-style-type: none"> • work with quantities in a contextual situation • use different representations for multiplication <p>Based on these, credit for specific aspects of performance should be assigned as follows</p>	
<p>1. Gives correct answer: \$ 48.00</p> <p><i>Shows work such as: (award one point for each correct solution strategy)</i></p> <p>$4 \times \\$12.00 = \\48.00</p> <p>$\\$12.00 + \\$12.00 + \\$12.00 + \\$12.00 = \\$48.00$</p> <p><i>(Accept any correct solution strategy including concrete, number line, and/or bar/area models.)</i></p>	<p>1</p> <p>2x1</p>
<p>2. Gives correct answer: 6 rows</p> <p><i>Shows work such as:</i></p> <p>$30 \div 5 = 6$ and/or</p> <p><i>(Accept any correct solution strategy including concrete, number line, and/or bar/area models.)</i></p>	<p>1</p> <p>2x1</p>
<p>3. See <u>Answer Key</u> on next page showing multiple representations/models for this problem.</p> <p>7 or 8 correct answers</p> <p>5 to 6 correct answers</p> <p>4 to 5 correct answers</p> <p>3 correct answers</p>	<p>4 pts</p> <p>3 pts</p> <p>2 pt</p> <p>1 pt</p>
Total Points	10

Answer Key for #3**Baking Cookies**

Answers may vary.....

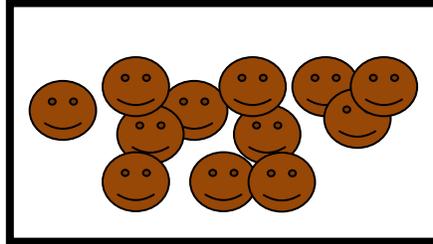
$\square \times 6 = 24$	$4 \times \square = 24$	$4 \times 6 = \square$
<p>Jill baked 24 cookies for her friends. She had 6 cookies in each row in her pan, How many rows of cookies were in the pan?</p>	<p>Jill baked 24 cookies for her friends. She had 4 rows of cookies in each pan. How many cookies were in each row?</p>	<p>Jill baked cookies for her friends. She had 4 rows of 6 cookies in the pan How many cookies did she bake?</p>
		
<p style="text-align: center;">6</p> <div style="border: 1px solid black; width: 100px; height: 50px; margin: 0 auto; display: flex; align-items: center; justify-content: center;"> ? 24 </div>	<p style="text-align: center;">?</p> <div style="border: 1px solid black; width: 100px; height: 50px; margin: 0 auto; display: flex; align-items: center; justify-content: center;"> 4 24 </div>	<p style="text-align: center;">6</p> <div style="border: 1px solid black; width: 100px; height: 50px; margin: 0 auto; display: flex; align-items: center; justify-content: center;"> 4 ? </div>

Student Materials

- **Pre/Post Assessment**

Performance Task

Baking Cookies



1. Jill is baking “Happy Face” cookies for 4 of her special friends. If she wants each friend to have 12 cookies, how many cookies will she need to bake?

_____cookies

Show how you got your answer.

Show a different way to get your answer.

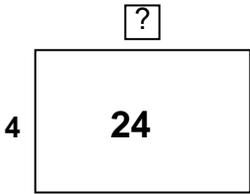
2. Jill decides to give each of her 30 classmates a cookie. If she can fit 5 cookies in a row on her baking pan, how many rows of cookies will she need to make?

_____rows

Show how you got your answer in two different ways.

3. Jill decides to only bake cookies for her **4** special friends. She is using a pan that holds **4 rows of 6 cookies or 24 cookies total**. Let's pretend that **one piece** of this information is missing, and you have to solve the problem for that missing number.

In each **column** of the chart below, a different piece of this information is missing, and this is the number you have to find. Each **row** of the chart is a different representation. If the word problem is missing, make up a problem to match the clue given. If a different representation is missing, use the clues in the chart below to fill in the other representations.

Equation		$4 \times 6 = \square$
Word Problem		<p>Jill baked 24 cookies for her friends. She had 4 rows of cookies in each pan. How many cookies were in each row?</p>
Concrete Model		
Area Model		

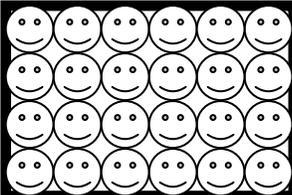
Rubric

Baking Cookies	
<p>The core elements of performance required by this task are:</p> <ul style="list-style-type: none"> • work with quantities in a contextual situation • use of different representations for multiplication <p>Based on these, credit for specific aspects of performance should be assigned as follows</p>	
<p>1. Gives correct answer: \$ 48.00</p> <p><i>Shows work such as: (award one point for each correct solution strategy)</i></p> <p>$4 \times \\$12.00 = \\48.00</p> <p>$\\$12.00 + \\$12.00 + \\$12.00 + \\$12.00 = \\$48.00$</p> <p><i>(Accept any correct solution strategy including concrete, number line, and/or bar/area models.)</i></p>	<p>1</p> <p>2x1</p>
<p>2. Gives correct answer: 6 rows</p> <p><i>Shows work such as:</i></p> <p>$30 \div 5 = 6$ and/or</p> <p><i>(Accept any correct solution strategy including concrete, number line, and/or bar/area models.)</i></p>	<p>1</p> <p>2x1</p>
<p>3. See <u>Answer Key</u> on next page showing multiple representations/models for this problem.</p> <p>7 or 8 correct answers</p> <p>5 to 6 correct answers</p> <p>4 to 5 correct answers</p> <p>3 correct answers</p>	<p>4 pts</p> <p>3 pts</p> <p>2 pt</p> <p>1 pt</p>
Total Points	10

Answer Key

Baking Cookies

Answers may vary.....

$\square \times 6 = 24$	$4 \times \square = 24$	$4 \times 6 = \square$
<p>Jill baked 24 cookies for her friends. She had 6 cookies in each row in her pan, How many rows of cookies were in the pan?</p>	<p>Jill baked 24 cookies for her friends. She had 4 rows of cookies in each pan. How many cookies were in each row?</p>	<p>Jill baked cookies for her friends. She had 4 rows of 6 cookies in the pan How many cookies did she bake?</p>
		
<p style="text-align: center;">6</p> <div style="border: 1px solid black; width: 100px; height: 100px; display: flex; align-items: center; justify-content: center;"> ? 24 </div>	<p style="text-align: center;">?</p> <div style="border: 1px solid black; width: 100px; height: 100px; display: flex; align-items: center; justify-content: center;"> 4 24 </div>	<p style="text-align: center;">6</p> <div style="border: 1px solid black; width: 100px; height: 100px; display: flex; align-items: center; justify-content: center;"> 4 <div style="border: 1px solid black; padding: 5px; display: flex; align-items: center; justify-content: center;"> ? </div> </div>

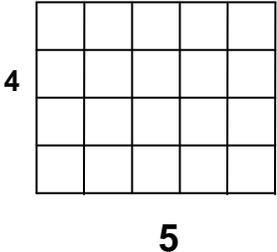
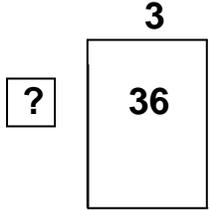
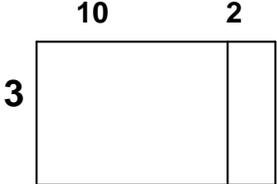
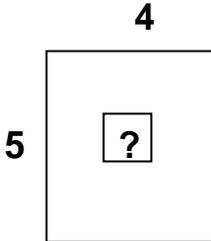
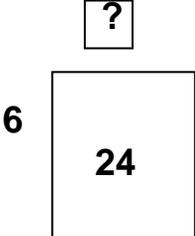
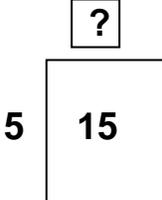
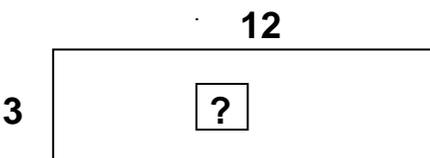
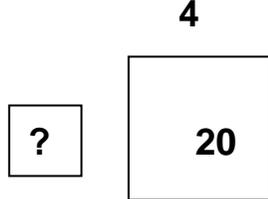
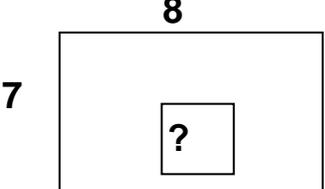
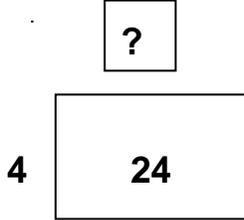
Card Set A-Equations

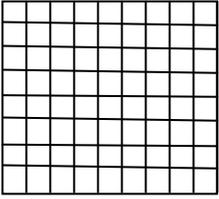
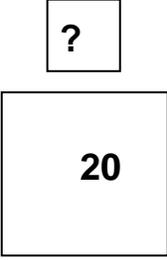
E1 $20 \div \square = 4$	E2 $7 \times 8 = \square$
E3 $5 \times \square = 15$	E4 $24 \div 4 = \square$
E5 $4 \times 5 = \square$	E6 $8 \times 9 = \square$
E7 $6 \times \square = 24$	E8 $20 \div 5 = \square$
E9 $15 \div 5 = \square$	E10 $3 \times 12 = \square$
E11 $5 \times 4 = \square$	E12 $\square + \square = 36$
E13 $36 \div \square = 3$	E14

Card Set B – Explanations in Words

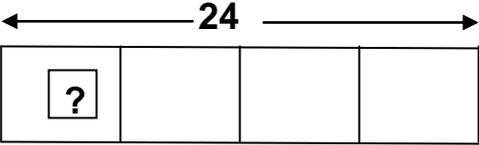
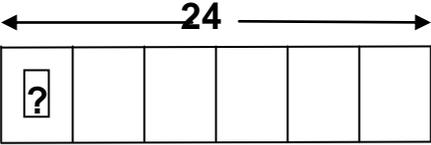
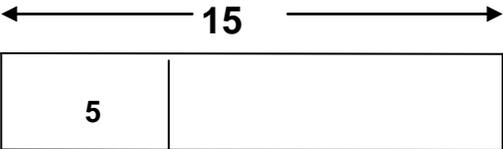
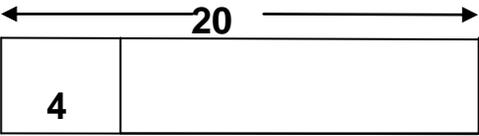
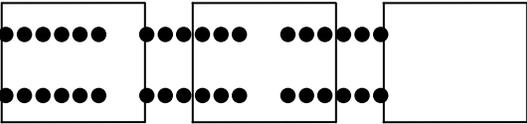
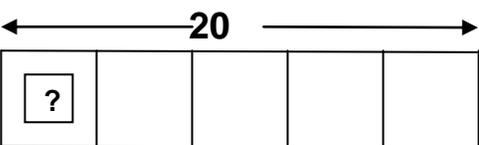
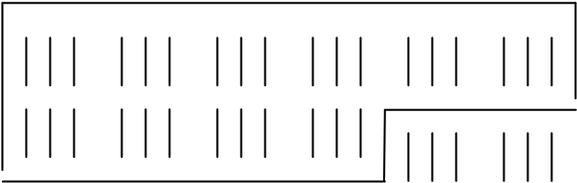
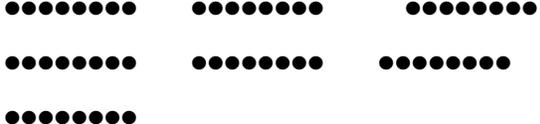
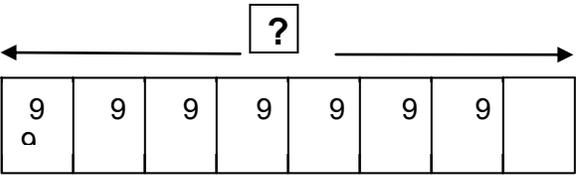
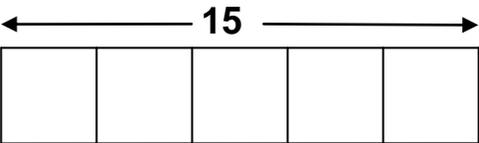
W1 4 times as big as 5	W2 5 groups of 4
W3 10 groups of 3 and 2 groups of 3	W4 3 groups of 12
W5 <input type="text"/> number of equal groups of 3 in 15	W6 5 equal groups of <input type="text"/> in 15
W7 <input type="text"/> number of equal groups of 4 in 20	W8 20 divided into 5 equal groups
W9 7 groups of 8	W10 8 times as big as 9
W11 6 groups of <input type="text"/> is 24	W12 24 is divided into 4 equal groups of <input type="text"/>
W13 36 is divided into 3 equal groups of <input type="text"/>	W14

A1 Card Set C - Area Model

<p>AM1</p> 	<p>AM2</p> 
<p>AM3</p> 	<p>AM4</p> 
<p>AM5</p> 	<p>AM6</p> 
<p>AM7</p> 	<p>AM8</p> 
<p>AM9</p> 	<p>AM10</p> 

<p>AM 11</p> <p>8</p> <p>9</p> 	<p>AM12</p> <p>?</p> <p>5</p> <p>20</p> 
<p>AM13</p>	<p>AM14</p>

Card Set D –Concrete Models

<p>CM1</p> 	<p>CM2</p> 
<p>CM3</p>  <p style="text-align: center;">? groups of 5</p>	<p>CM4</p>  <p style="text-align: center;">? groups of</p>
<p>CM5</p> 	<p>CM6</p> 
<p>CM7</p> 	<p>CM8</p> 
<p>CM9</p> 	<p>CM10</p> 

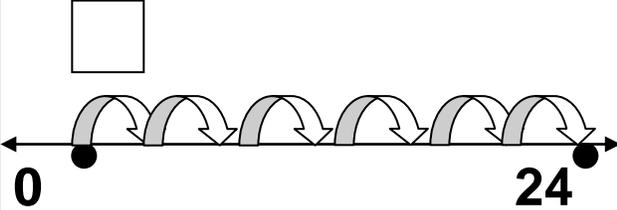
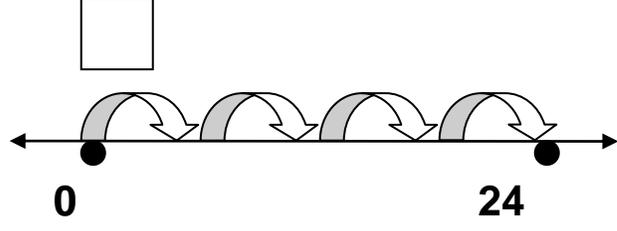
CM11 	CM12
CM13	CM14

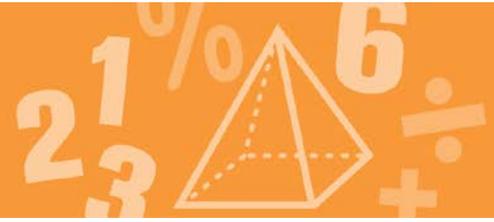
Card Set E – Contextual Problems

<p>W1 Joe has 20 crayons. How many crayons does he put in each box if he gets 4 boxes for 20 crayons?</p>	<p>W2 Susie wants to give her 7 friends 8 pieces of candy each. How many pieces of candy will she need to buy?</p>
<p>W3 Polly’s mom has planted 15 plants in 5 rows. How many plants are there in each row?</p>	<p>W4 Sam’s dad bought 24 hotdogs for Sam and his 3 friends. How many hot dogs can they each have?</p>
<p>W5 Debbie wants to give her 5 friends 4 balloons each. How many balloons must she buy?</p>	<p>W6 Manson wants to give his 8 friends 9 baseball cards each. How many baseball cards must he buy?</p>
<p>W7 Sarah buys 6 pieces of bubble gum for 24 cents. How much does each piece cost?</p>	<p>W8 Caitlin buys 5 pies for \$20 each. How much does one pie cost?</p>
<p>W9 Daniel buys 3 dozen donuts. How many donuts does he buy?</p>	<p>W10 David wants to give his 4 friends 5 books each. How many books will he need?</p>
<p>W11</p>	<p>W12 Mardi plants 36 plants in 3 rows. How many plants in each row?</p>

Card Set F - Number Line Model

<p>NL 1</p> <p>5 5 5 5</p> <p>0 <input type="text"/></p>	<p>NL 2</p> <p>4 4 4 4 4</p> <p>0 <input type="text"/></p>
<p>NL 3</p> <p><input type="text"/></p> <p>0 <input type="text"/> 36</p>	<p>NL 4</p> <p>12</p> <p>0 <input type="text"/></p>
<p>NL 5</p> <p>5 <input type="text"/> ? groups of 5</p> <p>0 15</p>	<p>NL 6</p> <p><input type="text"/></p> <p>0 15</p>
<p>NL 7</p> <p>5 <input type="text"/> ? groups of 5</p> <p>0 20</p>	<p>NL 8</p> <p><input type="text"/></p> <p>0 20</p>
<p>NL 9</p> <p>8 8 8 8 8 8 8</p> <p><input type="text"/></p>	<p>NL 10</p> <p>9 9 9 9 9 9 9 9</p> <p><input type="text"/></p>

<p>W11</p> <p><input type="text"/></p>  <p>A horizontal number line with arrows at both ends. It is labeled with '0' at the left end and '24' at the right end. There are two solid black dots on the line, one at 0 and one at 24. Six identical curved arrows are drawn above the line, each starting from a point and ending at a point to its right, representing equal jumps from 0 to 24.</p>	<p>W12</p> <p><input type="text"/></p>  <p>A horizontal number line with arrows at both ends. It is labeled with '0' at the left end and '24' at the right end. There are two solid black dots on the line, one at 0 and one at 24. Four identical curved arrows are drawn above the line, each starting from a point and ending at a point to its right, representing equal jumps from 0 to 24.</p>
<p>W13</p>	<p>W14</p>



GRADE 3 MATH: COOKIE DOUGH

EXPERT INVESTIGATION: SQUIRRELING IT AWAY



Expert Task

Squirreling It Away



Level A:

Austin has a bag of 17 acorns. Eight squirrels came up to him. He gave each squirrel an acorn. Then five more squirrels came up to him and he gave away one acorn to each of them. How many more squirrels can he still feed?

Show how you figured it out.

How do you know you have the right answer?



Expert Task

Squirreling It Away



Level B:

Austin likes to watch squirrels find and store acorns for the winter. Brown Squirrels can carry two acorns at a time. Gray Squirrels can carry three acorns at a time and Black Squirrels can carry five acorns at a time. There is a pile of 24 acorns.

How many trips would a Brown Squirrel need to make to store all of the acorns in the pile?

How many trips would a Gray Squirrel need to make to store all of the acorns in the pile?

How many trips would a Black Squirrel need to make to store all of the acorns in the pile?

If all three squirrels worked together to store the acorns how many trips would the squirrels need to make to store all of the acorns?

Explain your solution.



Expert Task

Squirreling It Away



Level C:

Brown Squirrels can carry 2 acorns at a time.
Gray Squirrels can carry 3 acorns at a time.
Black Squirrels can carry 5 acorns at a time.

Suppose all three squirrels want to store acorns for the winter. Depending on how motivated each squirrel is they would end up with different amounts. For instance, suppose the Brown Squirrel took 4 trips, the Gray Squirrel took 2 trips, and the Black Squirrel took 2 trips. The Brown Squirrel would end up with 8 acorns, the Gray Squirrel would have 6 acorns and the Black Squirrel would have 10. If between them all they took every one of the 24 acorns:

How many different ways could the three squirrels divide up the 24 acorns and not leave any left over? Each squirrel must carry their maximum load on each trip.

How do you know that you have found all of the ways?



Expert Task

Squirreling It Away



Level D:

The squirrels are rather smart. They realize that they can carry less than their maximum loads. How many different ways could the squirrels divide up the 24 acorns?

Explain your solution.



GRADE 3 MATH: COOKIE DOUGH SUPPORTS FOR ENGLISH LANGUAGE LEARNERS

GRADE 3 MATH: COOKIE DOUGH

Supports for ELLs

Title: Cookie Dough	Grade: 3
Linguistic Access: <p>In these supportive materials, a distinction between the vocabulary and the language functions is needed to provide entry points to the math content. Both need to be clarified to ensure comprehension and to avoid misunderstanding. This can be done by introducing and/or reviewing the most essential vocabulary and language functions in context and with concrete models, when applicable, in order for English Language Learners (ELLs) to better understand the meaning of the terms. The following vocabulary/language functions are suggested:</p> <p><u>Vocabulary Words/Phrases</u></p> <p>Tier I (non-academic language): raise, fundraising, sponsored (walk), classmates, tub, lap, efficient, store, maximum</p> <p>Tier II (general academic language): at least, row, column, equal groups, time, solution,</p> <p>Tier III (math technical language and concepts that must be carefully developed): multiple, product, quotient, divide, factor, array</p> <p><u>Language Functions:</u> figure, decide, analyze, explain, estimate</p>	
Content Access: <p>To provide content access to ELLs, it is important that they are familiar with the concept of creating diagrams to illustrate a situation or a pattern in order to see the relation between multiplication and repeated addition. Students should be familiar with how to organize information into an array, as well as understand the area model for multiplication and be able to explain the number line representation.</p> <p>In order to successfully develop the concept of multiplication, different models must be presented to ELL students. In addition to understanding the concept of an “array” and the area model as it is presented in the context of the card sets in the performance task “Baking Cookies” (see pages 44–56), ELLs must visualize the “equal groups.” The relationship between addition and subtraction must be reinforced before the relationship between multiplication and division is introduced.</p>	
Scaffolds and Resources: <ul style="list-style-type: none">• Organize tasks to maximize opportunities for ELLs to engage in math discourse. It is recommended that:	

- Teacher plans for students to work collaboratively in pairs or triads.
 - Teacher allows ELLs to use their language resources, including their native language, gestures, drawings, etc., to convey their understandings.
 - Teacher models not only math content but also the desired academic language in context to develop students' academic discourse. Reinforce the expression "equal groups" when explaining multiplication, for example.
 - Teacher uses paraphrases and "re-voicing" (reformulation of students' statements using appropriate math terminology or syntax).
 - Teacher uses physical objects to facilitate students' talk (e.g., manipulatives such as color tiles, base 10 materials, cm grid paper, and calculators).
 - Teacher provides an appropriate wait time for ELLs to respond.
 - Teacher gives the opportunity for ELLs to clarify their statements using different expressions.
- There are some instructional implications that teachers can use to scaffold ELLs' knowledge while introducing the concepts in the tasks:
 - Reinforce the concept of inverse operations as it relates to multiplication.
 - Develop the understanding of the identity element (1) for multiplication.
 - Develop the understanding of "0" as a factor in multiplication.
 - Provide opportunities for students to multiply by 1, 10, and 100.
 - Use the calculator to help ELLs understand what occurs in the quotient as the divisor gets smaller (e.g., $12/12$, $12/6$, $12/4$, $12/3$, $12/2$, $12/1$, and then 12 divided by one half). Teacher can ask students: "As you continue the pattern and the divisor approaches 0, what is happening to the quotient?"



GRADE 3 MATH: COOKIE DOUGH SUPPORTS FOR STUDENTS WITH DISABILITIES

GRADE 3 MATH: COOKIE DOUGH

INSTRUCTIONAL SUPPORTS FOR STUDENTS WITH DISABILITIES USING UDL GUIDELINES

Foundational Information for Educators

Universal Design for Learning (UDL) is a framework that acknowledges student difference and diversity in how they comprehend, navigate, and interact with the world, and thus, responds by flexibly adapting curriculum, instruction, and the environment to meet student needs. Critical to fulfilling this agenda is the aim to reduce or eliminate barriers and increase access to learning for all students. Within this framework, UDL empowers educators to meet students' distinct and diverse needs by establishing and using "flexible goals, methods, materials, and assessments" (CAST, 2011, p. 4). Supporting this approach, three, overarching principles—each based upon advanced knowledge in neuroscience—and supporting checkpoints serve as the essential context from which educators enhance student motivation, access to content, and expression of understanding.

The first principle suggests that information should be represented using multiple means of presentation. Accordingly, Principle 1 suggests that "learners differ in the ways that they perceive and comprehend information that is presented to them" (CAST, 2011, p. 5) and should therefore, have the opportunity to encounter the content imparted through varying auditory and visual modes. Principle 2 recommends that educators afford students the opportunity to act upon, respond to, and express what they know through varying modalities. Hence, students differ in their facility to strategically plan and plot a course for accessing and extrapolating meaningful knowledge from the curriculum, and thus "require a great deal of strategy, practice, and organization" (CAST, 2011, p. 5) in order to succeed. Finally, Principle 3 states that students differ in their levels of drive and affect for learning and that accounting for and addressing varying factors that contribute to these differences, increasing students' motivation to learn and self-regulation skills provides students with measures for gauging and facilitating learning.

The framers of UDL embrace the philosophy that all learners can become expert learners. Reflecting and illuminating each of the principles, the UDL framework maintains that expert learners are (a) resourceful and knowledgeable learners who bring considerable prior knowledge to new learning, and activate that prior knowledge to identify, organize, prioritize, and assimilate new information; (b) strategic and goal-directed and formulate plans for learning and devise effective strategies and tactics to optimize learning; and (c) purposeful and motivated, eager for new learning, and motivated by the mastery of learning itself (CAST, 2011, pp. 6-7). Recommendations, henceforth, related specifically to this grade 5 set of tasks, advance practical ways to operationalize UDL concepts for meeting the needs of all learners.

See the following links to learn more about UDL and how to flexibly tailor curriculum, instruction, materials, and methods to ensure learning for all students:

www.cast.org

<http://www.udlcenter.org/aboutudl/udlguidelines>

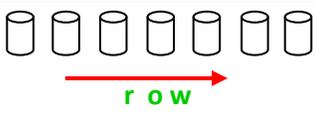
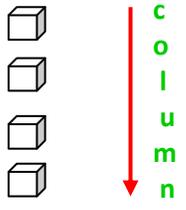
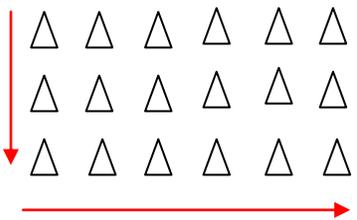
Anchor Concepts through Pre-Teaching: Multiplication and the Addition and Multiplication Relationship

The Cookie Dough Unit requires students to transition from relying on the more basic skills of recalling and recognizing and using procedures to employing the higher level reasoning skills of explaining, concluding and making connections. The instructional strategy progressing from concrete (hands-on, physical models or manipulatives to represent numbers and unknowns) to representational (semi-concrete, pictorial representations of the models) to abstract (numbers as abstract symbols or pictorial displays) has proven to be effective with students with disabilities. The higher level skills, generally, prove to be more difficult for students with disabilities to access and require explicit instruction and strategic guided practice.

The big idea of the unit requires students to be able to understand the meaning of multiplication and its inverse relationship to division. Foundational, however, is students' understanding of the relationship between addition and multiplication. Essentially, building conceptual understanding of multiplication and division is at the foundation of integrating the use of memorized facts and processes for calculating and solving problems. The internet is a resource for many free, interactive software students can access and navigate, easily. References are included, below.

- **Use multiple media for expression and communication.** Incorporate the use of physical manipulatives and interactive web tools to build conceptual understanding.
- **Use multiple tools for construction and composition.** Provide use of calculator to demonstrate efficiency of multiplication and division.
- **Clarify vocabulary and symbols.** Pre-teach relevant vocabulary related to the operations and the relationships between the operations.
- **Activate or supply background knowledge and optimize relevance, value, and authenticity.** Recruit students' interest by associating the application of mathematics and problem solving to real life situations of value to the learners. Begin with and build upon concepts and topics students are already familiar before introducing concepts involving more complex processes and understanding.
- **Maximize transfer and generalization.** Students are required to know when and how to apply multiplication for solving problems. One approach to ensuring students are able to understand the concept of multiplication is to use objects and visuals to depict the relationship between multiplication and addition. The goal is that students will be able to apply multiplication as a means to adding groups or sets of numbers more efficiently. In other words, they will be able to transfer their knowledge of the relationship between the two concepts in problem solving.
- **Offer ways of customizing the display of information.** Include multi-sensory modes of presenting concepts that include the incorporation of the visual, auditory, tactile-kinesthetic modalities. Use visual representations, such as drawings and manipulatives or concrete items to scaffold understanding of the operational, abstract concepts.
- **Foster collaboration and community.** Engage students in cooperative learning activities that will require them to collectively problem solve, communicate ideas, and represent concepts.

Multiplication Vocabulary

Term	Definition	Example
Multiplication	The act of adding a number to itself a particular number of times.	How many balls?  $3 + 3 + 3 + 3 + 3 = 15$ or $3 \times 5 = 15$
Product	The answer in multiplication.	$5 \times 8 = 40$ <small>product</small>
Factor	Numbers that are multiplied give a product.	$5 \times 8 = 40$ <small>factor factor</small>
Multiple	The result of multiplying by a whole number.	$4 \times 6 = 24$ 24 is a multiple of 4 and 6.
Rows	Series of items placed next to each other in a straight line, or horizontally.	
Column	Items placed one above the other, or vertically.	
Array	Objects or symbols displayed in both rows and columns.	

Sample Activities:

Relationship between Addition and Multiplication in Equal Groups & Equal Rows

Initiate exploration of the relationship between addition and multiplication using physical objects that can be manipulated and grouped into sets. Using situational problems, as a starting point, demonstrate and model the thinking process for sorting objects into groups and determining the answers to those real problems. Engaging students in graphic and visually organized examples, such as those that follow is the next step after, or depending on the child’s accessibility to manipulatives, in lieu of grouping tangible objects. Use the term “group” interchangeably with “set”.

How Many?

Running on the Beach



Ms. Martin’s showed the class a video of horses running on the beach. The owner wanted to buy them horse shoes. How many horse hooves would need horse shoes?

$4 + 4 + 4 + 4 + 4 + 4 + 4 = 28$ horse hooves

$4 \times 7 = 28$

What’s the pattern?

Running at the Track



Ms. Martin’s students are running in the school’s Olympic Fest before they make cookies. She wants to buy special bands for everybody’s arms. How many arms?

$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 24$ arms

$2 \times 12 = 24$

What’s the pattern?

How Many & What is the Pattern?

How many cookies?



$$5 + 5 + 5 + 5 = 20 \quad 4 + 4 + 4 + 4 + 4 = 20$$

or

$$5 \times 4 = 20 \quad 4 \times 5 = 20$$

How many Fancy Fruit?



$$3 + 3 + 3 + 3 = 12 \quad 4 + 4 + 4 = 12$$

or

$$3 \times 4 = 12 \quad 4 \times 3 = 12$$

What's the Pattern?

How many statues of students in class?



$$5 + 5 + 5 + 5 + 5 + 5 = 35 \quad 7 + 7 + 7 + 7 + 7 = 35$$

or

$$5 \times 7 = 35 \quad 7 \times 5 = 35$$

How many containers of peanut butter on the 2, bottom rows?

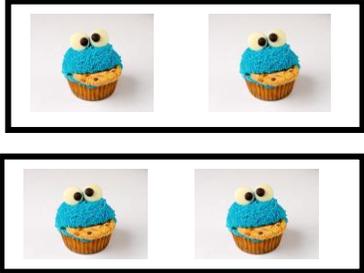
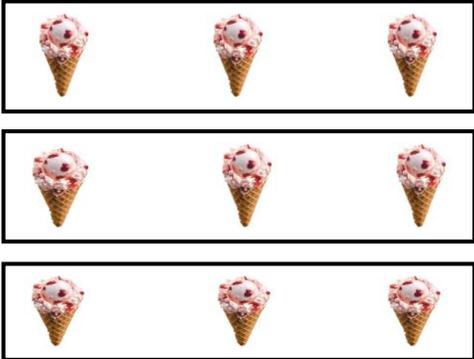
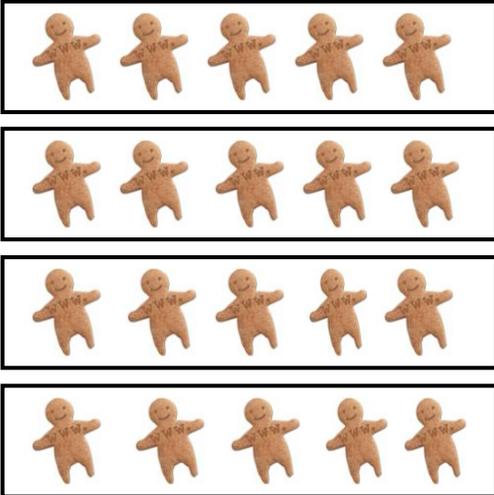


$$9 + 9 = 18 \quad 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 18$$

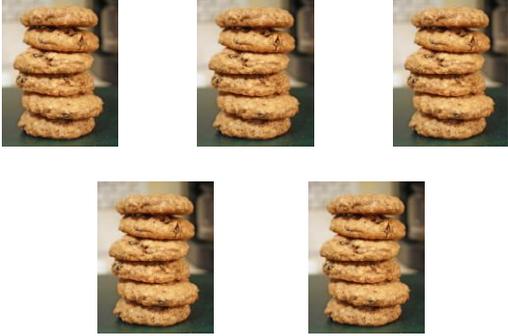
or

$$9 \times 2 = 18 \quad 2 \times 9 = 18$$

How Many & What is the Pattern?

Circle the Groups	How many groups?	How many are in each group?	How many altogether?
			
			
			

How Many Cookies?

Groups	Total	Adding	Multiplying
 <p>Two groups of cookies. Each group has 3 cookies.</p>	6	$3 + 3 = 6$ 3 cookies plus 3 cookies = 6 cookies	$2 \times 3 = 6$ 2 groups of 3 cookies each = 6
	16	$4 + 4 + 4 + 4 = 16$ Four cookies with cream, plus four cookies with cream, plus four cookies with cream, plus four cookies with cream = 16 cookies with cream	$4 \times 4 = 16$ 4 groups of 4 cookies with cream each = 16
	30	$6 + 6 + 6 + 6 + 6 = 30$ 6 oatmeal raisin cookies, plus 6 oatmeal raisin cookies = 30 oatmeal raisin cookies	$5 \times 6 = 30$ 5 groups of 6 oatmeal raisin cookies = 30 oatmeal raisin cookies.

What's the pattern?

Figure Out How Many

A Real Problem	One Way to Figure It Out	Explain What You Did
 <p>Jamal is having a birthday party in her classroom. <u>Everyone is eating cookies.</u> The class has <u>5 tables</u> with <u>4 students</u> at <u>each table.</u> <u>How many students are eating cookies?</u></p>	<p>Table 1 ☺ ☺ ☺ ☺</p> <p>Table 2 ☺ ☺ ☺ ☺</p> <p>Table 3 ☺ ☺ ☺ ☺</p> <p>Table 4 ☺ ☺ ☺ ☺</p> <p>Table 5 ☺ ☺ ☺ ☺</p> <p><u>4 students</u> + <u>4 students</u> + <u>4 students</u> + <u>4 students</u> + <u>4 students</u> = 20 students eating cookies</p>	<p>I underlined important facts in the problem. Then, I drew a picture with 5 tables and 4 kids at each table. I added all of the kids at all of the tables. There are 20 kids eating cookies.</p>
 <p>Jamal's mom bought enough milk for all 20 students in her class. She paid the lunch room lady \$2 for each glass. How much did she pay? Use addition.</p>		

A Real Problem	Another Way to Figure It Out	Explain What You Did
 <p>Mary is having a birthday party in her classroom. <u>Everyone is eating cookies.</u> The class has <u>5 tables</u> with <u>4 students</u> at <u>each table.</u> <u>How many students are eating cookies?</u></p>	<p><u>5 tables</u> x <u>4 kids</u> = 20 kids eating cookies</p> <p>or</p> <p><u>5 x 4</u> = 20 kids eating cookies</p>	<p>I underlined important facts in the problem. Since the same number of kids is sitting at each table, I just multiplied the number of kids, times the number of tables. I figured out 20 kids are eating cookies.</p>
 <p>Mary's mom bought enough milk for all 20 students in her class. She paid the lunch room lady \$2 for each glass. How much did she pay? Use multiplication.</p>		

How Many Strawberries?



Imani is going strawberry picking with her class. She wants to pick enough strawberries for everyone in her family. Including her mother and father, Imani has two brothers and one sister. How many strawberries does she have to pick if she wants everyone to receive 10 each?

One way to solve:

5	Mom	X	X	X	X	X	X	X	X	X	X
	Dad	X	X	X	X	X	X	X	X	X	X
	Ade	X	X	X	X	X	X	X	X	X	X
	Adisa	X	X	X	X	X	X	X	X	X	X
	Ife	X	X	X	X	X	X	X	X	X	X

10 strawberries each

$10 + 10 + 10 + 10 + 10 = 50$ strawberries altogether

(10 strawberries for Mom + 10 strawberries for Dad + 10 strawberries for Ade + 10 strawberries for Adisa + 10 strawberries for Ife equals 50 strawberries altogether)

Another way to solve:

5 family members x 10 strawberries each = 50 strawberries, altogether

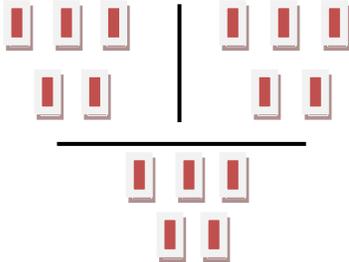
Another way to solve:

$5 \times 10 = 50$ strawberries altogether

Anchor Concepts through Pre-Teaching: Division and the Multiplication and Division Relationship

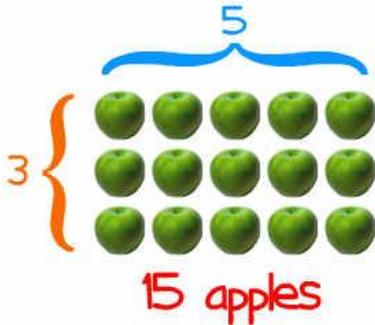
- **Offer ways of customizing information.** Adjust the size, font, color of text, graphs, visual content, and images to increase student visual access to information.
- **Use multiple media for expression and communication.** Incorporate the use of physical manipulatives and interactive web tools to build conceptual understanding of division and its relationship to multiplication.
- **Use multiple tools for construction and composition.** Provide use of calculator to demonstrate efficiency of multiplication and division.
- **Clarify vocabulary and symbols.** Pre-teach relevant vocabulary and mathematical symbols, such as division and equal signs that are directly related to division. Clarify any terms explaining the inverse relationship between division and multiplication.
- **Activate or supply background knowledge and optimize relevance, value, and authenticity.** Recruit students' interest by associating the application of problem solving involving multiplication and division to real life situations of value to the learners.
- **Offer ways of customizing the display of information.** Include multi-sensory modes of presenting concepts that include the incorporation of the visual, auditory, tactile-kinesthetic modalities. Use visual representations, such as drawings and manipulatives or concrete items to scaffold understanding of the operational, abstract concepts of division.
- **Foster collaboration and community.** Engage students in cooperative learning activities that will require them to collectively problem solve, communicate ideas, and represent concepts.

Division Vocabulary

Division Term	Definition	Example
Division	Splitting in equal parts or groups.	<p>Mike has 15 new baseball cards he wants to share them with his two friends. How does he split them three ways?</p>  <p>The ÷ symbol means divide. $15 \div 3 = 5$ <i>(15 divided by 3 equals 5)</i></p>
Dividend	The amount to be divided.	<p>In $15 \div 3 = 5$, or</p> $\begin{array}{r} 5 \\ 3 \overline{) 15} \end{array}$ <p>15 is the dividend.</p>
Divisor	The number you divide by (or the number of sets you divide the dividend).	<p>In $15 \div 3 = 5$, or</p> $\begin{array}{r} 5 \\ 3 \overline{) 15} \end{array}$ <p>3 is the divisor.</p>
Quotient	The answer after you divide one number into another.	<p>In $15 \div 3 = 5$,</p> $\begin{array}{r} 5 \\ 3 \overline{) 15} \end{array}$ <p>5 is the quotient.</p>
<p>dividend ÷ divisor = quotient</p> $\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \end{array}$		

Sample Activities: Relationship between Multiplication and Division

Let's figure out how many!



How many apples did Mark pick on the apple-picking trip?

$$3 \times 5 = 15 \text{ apples}$$

(3 apples times 5 apples equals 15 apples)

How many apples can Mark share with his 4 friends? He wants to keep some for himself, also.

$$15 \div 5 = 3 \text{ apples each}$$

(15 apples divided by 5 kids equals 3 apples each)

Mark is thinking about giving away all of his apples. He would like to give them to the principal, his teacher, and his mother. How many will each person get?

$$15 \div 3 = 5 \text{ apples each}$$

(15 apples divided by 3 adults equals 5 apples each)

Multiplication and Division Families

$$5 \times 3 = 15$$

$$15 \div 5 = 3$$

$$3 \times 5 = 15$$

$$15 \div 3 = 5$$

$$4 \times 4 = 16$$

$$16 \div 4 = 4$$

$$2 \times 8 = 16$$

$$16 \div 2 = 8$$

$$8 \times 2 = 16$$

$$16 \div 8 = 2$$

Let's figure out how many!



Cynthia has 4 packages of American dolls. How many dolls does she have?

$$4 \times 4 = 16 \text{ dolls}$$

(4 packs times 4 dolls each pack equals 16 dolls)

Cynthia will have 7 friends over for her sleepover party. She wants everyone, including herself, to play with dolls. How many dolls can each girl play with?

$$16 \div 8 = 2 \text{ dolls each girl}$$

(16 dolls divided by 8 girls equals 2 dolls each)

Suppose Cynthia's mother says she has to divide the dolls equally between herself and her younger sister. How many dolls will she have to play with?

$$16 \div 2 = 8 \text{ dolls}$$

(16 dolls divided by 2 girls equals 8 dolls each)

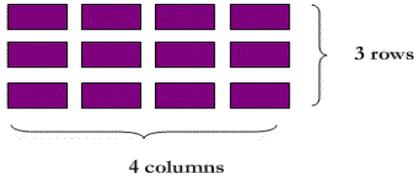
Suppose only 4 girls come to her sleepover. How many dolls will each girl play with?

$$16 \div 4 = 4 \text{ dolls each}$$

(16 dolls divided by 4 girls equals 4 dolls)

What's the Trick?

How Many Blocks?



Check!

Are the number of objects in each row the same? **Yes!**

Are the number of objects in each column the same? **Yes!**

Simple!

Multiply the number of rows times the number of columns! $3 \times 4 = 12$

What Do You Notice?

$$3 \times 4 = 12 \quad 12 \div 4 = 3 \quad \begin{array}{r} 3 \\ 4 \overline{)12} \end{array}$$

$$4 \times 3 = 12 \quad 12 \div 3 = 4 \quad \begin{array}{r} 4 \\ 3 \overline{)12} \end{array}$$

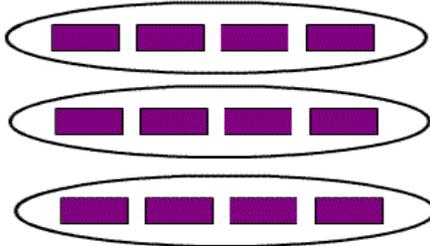
3 times 4 equals 12

4 times 3 equals 12

12 divided by 3 equals 4

12 divided by 4 equals 3

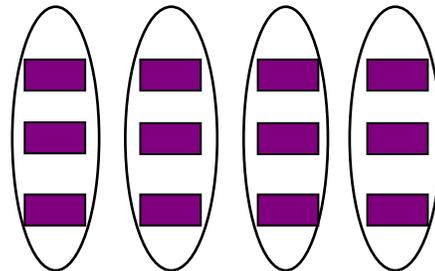
A Division and Multiplication Family



The blocks have been divided into 3 equal rows!

3 rows x 4 columns = 12 altogether

$12 \div 3$ rows = 4 in each row or
 $12 \div 3 = 4$

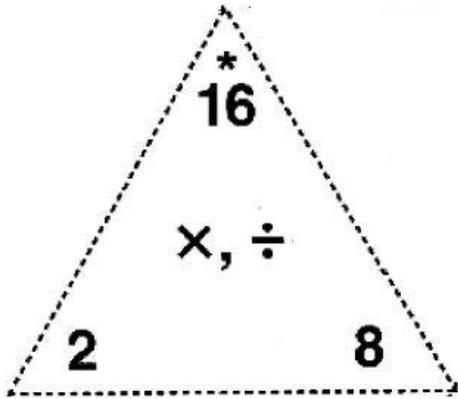


The blocks have been divided into 4 equal columns!

4 columns x 3 rows = 12 altogether

$12 \div 4$ columns = 3 in each columns or
 $12 \div 4 = 3$

An Example of a Division and Multiplication Family



Multiplication and Division Fact Triangle

MathATube.com

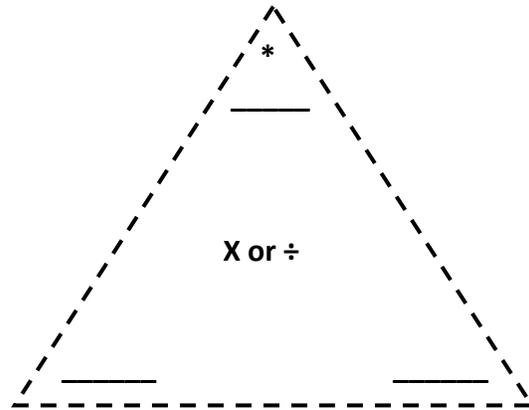
$2 \times 8 = 16$

$16 \div 8 = 2$

$8 \times 2 = 16$

$16 \div 2 = 8$

Complete the Division and Multiplication Fact Family for 6 and 7.



___ x ___ = ___	___ x ___ = ___
___ ÷ ___ = ___	___ ÷ ___ = ___

Draw 6×7 using objects or symbols in rows and columns.

Draw $42 \div 6$ using objects or symbols.

Draw $42 \div 6$ using objects or symbols.

Pre-Teach and Reinforce Skills for Processing Information

- **Clarify vocabulary and symbols.** Pre-teach relevant vocabulary related to the operations and the relationships between the operations. Introduce important words and phrases by engaging students in relevant and transferrable experiences related to the application of key operational and conceptual processes, such as terms that indicate when to multiply. Post and refer to guides when applicable.
- **Highlight patterns, critical features, big ideas, and relationships.** Expert learners can distinguish between important and inconsequential or irrelevant information. When attempting to gain understanding of mathematical problems, providing explicit prompts and cues assists novice learners with identifying and highlighting patterns, critical features, and important relationships.
- **Guide information processing, visualization, and manipulation.** Support learners with processing information by providing explicit, sequential, and step-by-step prompts. Provide options for organizing information; multiple strategies for solving problems; and varied ways for classifying, reinforcing methods for gathering and organizing facts.
- **Maximize transfer and generalization.** To strengthen students' memorability of information, utilize techniques that reinforce recall and application, such as checklists, organizers, mnemonic devices, and think-alouds that entail modeling and active demonstration.
- **Build fluencies with graduated levels of support for practice and performance.** Enhance students' fluency in mathematical problem solving and calculation by providing many opportunities that involve practice, scaffolded support and release to increase independence, review and discussion of multiple examples, and feedback tailored to meet the specific needs of the learner.
- **Support planning and strategy development.** Provide guided support, coaching, prompting mechanisms, and organizing templates to assist learners with acquiring and sustaining skills necessary for becoming intentional and strategic learners.
- **Enhance capacity for monitoring progress.** Foster students' self-assessment and monitoring behaviors by posing questions, checklists, or organizers that require students to self-reflect and self-correct. Ensure teacher feedback is timely, specific, informative, and accessible. Teach students to seek and obtain the type of assistance they need in order to facilitate understanding, organization, or advancement.

Key Math Operation Words and Phrases

Key Words for Solving Problems Involving Addition*	
Key Words/Phrases	Selected Examples
increased by	The price was increased by \$10.00.
more than	She had 5 dolls more than before. She had 5 more dolls than before.
combined	They combined many kinds of nuts in the trail mix.
together	Together they collected 27 sea shells.
total or total of	What is the total cost of Tamika’s gifts she bought for her family?
sum	He spent a sum of \$10 on marbles and baseball cards.
added to	Ian’s pay was added to his savings. Ian added his pay to his savings.
altogether	How many crayons do they have altogether ?
both	How much money did both teams raise for the celebration?
in all	How many party packs do we need in all ?
additional	How many legos will Nia have if she receives an additional 37?
all	For a birthday party for Ru and Yeo, how many candles do we need in all ?
another	If we include another bag of pencils, how many pencils will we have?

Adapted from Melissa S. Moreno’s “Key Words for Solving Word Problems” posted on www.Gateways2Learning.com.

Key Words for Solving Problems Involving Subtraction*	
Key Words/Phrases	Selected Examples
decreased by	Because the wind blew many away, the balloons were decreased by 35.
minus	Take my change minus \$1.50 for the donation. How much can you have?
except	Use all of the raisins in the recipe, except $\frac{1}{4}$ cup I’ll use in my oatmeal.
less	I gave Maria \$2.25 less the \$.75 she owed me. How much did she get?
difference between or difference of	What is the difference between Monica and Bonita’s height? What is the difference of me paying HG Discount store \$1.00 for 10 balloons or paying PJ Favorites \$1.20 for 10 balloons?
more than	Hazel has 14 more marbles than Shelton? How many does she have?
less than	Trevon has 6 baseball cards less than his older brother. How many does Trevon have?
fewer than	Mika’s team caught 7 fewer balls than Akil’s team.
How many more?	How many more invitations does Randy have to send, now that he’s increased his list?
How much more?	How much more water does Chan need to add to equal 17 cups?
left	How many passengers were left on the bus?
remain or remains	How many potted plants remain from our sale, today?
Words ending in “er”. Examples: higher, longer, faster, heavier, etc.	Who can jump higher? Sally or Jamila? How much longer is Dish’s shadow than Kwesi’s? How much faster did Li run than Jose?
take away	If Raul takes away 7 blocks, how many will he have?
only	If Amy take only 5 ribbons, how many will Heather have left?

Adapted from Melissa S. Moreno’s “Key Words for Solving Word Problems” posted on www.Gateways2Learning.com.

Key Words for Solving Problems Involving Multiplication*	
Key Words/Phrases	Selected Examples
of	Megan ate half of the cherries Nana bought. (multiplication by $\frac{1}{2}$)
times	Sizwe has 3 times as many family members as Miguel.
multiplied by	The animals in the shelter multiplied by 7 in the past 6 months.
product of	Kathryn wanted the number of beads equal to the product of 5 and 10.
factor of	The chickens increased by a factor of 8.
at this rate	At what rate does Shen's father drive from his house to Granpa's?
each	Each student will receive 7 pencils. How many pencils will the teacher need for her class?
doubled, tripled, quadrupled, etc.	The department store tripled the original cost of the sports jersey.
twice, three times, four times, etc.	There are five times less cars in the parking lot on Monday when compared to Saturday.

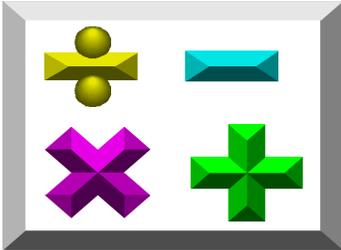
Adapted from Melissa S. Moreno's "Key Words for Solving Word Problems" posted on www.Gateways2Learning.com.

Key Words for Solving Problems Involving Division*	
Key Words/Phrases	Selected Examples
each	How many rings can you buy for \$5 if each one costs \$1? Rings cost \$1 each at the Flea Market. How many can I buy with \$5?
equal or equally	Benjamin divided his marbles equally into 6 bags. How many marbles were in each bag?
per	The speed limit on the highway is 55 miles per hour. How long will it take to drive from Selena's home to her aunt's house?
separate	Kofi's mom separated the shirts into 3 equal piles? How many shirts were in each pile?
a	Ice cream costs \$2 a scoop. How many scoops can you buy with \$10?
quotient or quotient of	What is the quotient of 27 divided by 3?
percent (division by 100)	What percent of students are absent today?

Adapted from Melissa S. Moreno's "Key Words for Solving Word Problems" posted on www.Gateways2Learning.com.

Schema-Based Strategy Instruction

Strategy developed by Jitendra, Griffin, Haria, Leh, Adams, and Kaduvetoor (2005), cited in Montague (2005), "Math problem solving for primary elementary students with disabilities", found at http://www.k8accesscenter.org/training_resources/mathprimaryproblemsolving.asp.

	<p>Find the problem type.</p>	
<p>Organize the information in the problem using a diagram.</p>		
	<p>Plan to solve the problem.</p>	
	<p>Solve the problem.</p>	

Problem-Solving Checklists and Strategies

“What Good Problem Solvers Do” Checklist

Did I:	How did I do it?	
READ the problem?	I reread it as many times as I needed to understand it.	
	I underlined key words.	
	I asked myself if I understood the problem.	
REWORDED or Paraphrased?	I put the problem in my own words?	
	I asked myself what the question is.	
	I asked myself what they are looking for.	
VISUALIZE or DRAW?	I drew a picture or diagram to show all parts of the problem. Or, I visualized an image of all parts of the problem.	
HYPOTHESE?	I thought of a sensible way to solve the problem.	
	I decided the operation or operations I would need to use.	
	I decided how many steps I would need to do.	
	I wrote the symbols where I need to use them.	
ESTIMATE?	I predicted the answer using mental math. Or, I estimated by rounding the numbers.	
COMPUTE?	I did the arithmetic for each step.	
	I compared my answer with my estimate.	
	I asked myself if the answer made sense.	
	I used the right symbols and labels.	
CHECK?	I checked to see that I calculated correctly and used the right signs and labels.	

Adapted from Montague (2005), “Math problem solving for primary elementary students with disabilities”, found at

http://www.k8accesscenter.org/training_resources/mathprimaryproblemsolving.asp.

4 Steps to Problem Solving*

<p>1. UNDERSTANDING THE PROBLEM</p> <ul style="list-style-type: none">* Can you state the problem in your own words?* What are you trying to find or do?* What are the unknowns?* What information do you obtain from the problem?* What information, if any, is missing or not needed?	<p>2. DEVISING A PLAN</p> <ul style="list-style-type: none">* Look for a pattern.* Examine related problems, and determine if the same technique can be applied.* Examine a simpler or special case of the problem to gain insight into the solution of the original problem.* Make a table.* Make a diagram.* Write an equation.* Use guess and check.* Work backward.
<p>3. CARRYING OUT THE PLAN</p> <ul style="list-style-type: none">* Carry out the strategy or strategies in step 2, and do any necessary actions or computations.* Check each step of the plan as you go ahead.* Keep an accurate record of your work.	<p>4. LOOKING BACK</p> <ul style="list-style-type: none">* Check the results in the original problem. (In some cases this will require a proof.)* Does your answer make sense when you look at the original problem? Is it reasonable?* Is there is another method of finding the solution.

* Adapted from 4-step process found at <http://teacher.scholastic.com/lessonrepro/lessonplans/steppro.htm>.

Problem-Solving Strategies*



- Make a table.
- Make an organized list.
- Look for a pattern.
- Guess and check.
- Draw a picture or graph.
- Make a model.
- Work backwards.
- Solve a simpler problem.

Developing Mathematical Thinking**

Do	Ask
Build student confidence	<ul style="list-style-type: none"> • <i>Why is that true?</i> • <i>How did you reach that conclusion?</i> • <i>Does that make sense?</i> • <i>Can you make a model to show that?</i>
Help students reason mathematically	<ul style="list-style-type: none"> • <i>Is that true for all cases? Explain.</i> • <i>How would you prove that?</i> • <i>What assumptions are you making?</i>
Check student progress	<ul style="list-style-type: none"> • <i>Can you explain what you have done so far? What else is there to do?</i> • <i>Can you think of another method that might have worked?</i> • <i>What do you notice when . . . ?</i> • <i>Why did you decide to organize your results like that?</i>
Encourage conjecturing	<ul style="list-style-type: none"> • <i>What would happen if . . . ? What if not?</i> • <i>Do you see a pattern? Can you explain the pattern?</i>
Promote problem solving	<ul style="list-style-type: none"> • <i>What do you need to find out?</i> • <i>What information do you have?</i> • <i>What strategies are you going to use?</i> • <i>What tools will you need?</i> • <i>What do you think the answer or result will be?</i>
Help students get un-stuck	<ul style="list-style-type: none"> • <i>How would you describe the problem in your own words?</i> • <i>What do you know that is not stated in the problem?</i> • <i>What facts do you have?</i> • <i>Would it help if you created a diagram? Make a table? Draw a picture?</i>
Encourage reflection	<ul style="list-style-type: none"> • <i>How did you get your answer?</i> • <i>Does your answer seem reasonable? Why? Why not?</i> • <i>What if you had started with . . . rather than . . . ?</i> • <i>What have you learned or found out today?</i>

* Adapted from outline posted at <http://library.thinkquest.org/25459/learning/problem/?tqI-iframe>.

** Adapted from PBS TeacherLine (2006), Developing Mathematical Thinking with Effective Questions. For the complete list, visit http://www.pbs.org/teacherline/resources/questionsheet_vma.pdf.

References

Multiplication Concepts

Burns, M. (2008). *Do the math*. Scholastic: USA.

Develop mathematical thinking using this checklist:

http://www.pbs.org/teacherline/resources/questionsheet_vma.pdf?cc=tlredir

Interactive Math Tools

Web-based, interactive math tools for teachers and students listed and linked from this site:

http://www.k8accesscenter.org/training_resources/MathWebResources.asp

Interactive, student-friendly math skill drills and practice:

http://www.ixl.com/promo?partner=google&phrase=Misc%20-%20Math-Specific%20K-8%20-%20aaastudy.com&gclid=CLT04sX22awCFQfd4AodL2_rFA

Problem Solving

Problem-solving strategies and principles to guide extending problem types and essential skills:

<http://library.thinkquest.org/25459/learning/problem/>

Step-by-step guidance with solving one-step and two-step multiplication problems:

http://www.homeschoolmath.net/teaching/md/understanding_word_problems.php

Colorful, interactive multiplication and division problem-solving activities that include prompting and feedback:

http://www.thinkingblocks.com/ThinkingBlocks_MD/TB_MD_Main.html

Math word problems by grade, concept, skill, steps, and operation:

<http://www.mathplayground.com/wordproblems.html>

Terms, Vocabulary, Definitions

Multiplication vocabulary and definitions, each with audible pronunciation and speech features:

<http://quizlet.com/562923/multiplication-vocabulary-3rd-grade-flash-cards/>

Multiplication and division terminology explained with visual illustrations:

<http://www.mscoleman.com/Math%20Files/Multiplication%20Division%20Vocabulary.pdf>

Interactive site visually explaining and representing basic division terms and the three ways to divide:

<http://www.wisc-online.com/objects/ViewObject.aspx?!ID=ABM601>

A table of key words and phrases accompanied by examples for each of the four mathematical operations:

<http://www.gateways2learning.com/Algebra/KeyWords4WordProbs.pdf>

An animated, interactive online math dictionary for students which explains over 600 common mathematical terms and math words in simple language:

<http://www.amathsdictionaryforkids.com/>

Math skill and concept topics includes definitions and examples:

<http://www.aaastudy.com/grade3.htm#topic96>