Achieve

TO: Barry Erwin, President and Chief Executive Officer, Council for a Better Louisiana

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DATE: March 1, 2016

SUBJECT: Achieve's Review of the final draft of the Louisiana Standards for Mathematics

Executive Summary

The purpose of this review is to examine the January 2016, draft of the Louisiana Standards for Mathematics (LSM) [1] to determine whether they are high-quality standards that prepare students, over the course of their K–12 education careers, for success in credit-bearing college courses and quality, high-growth jobs.

When evaluating standards, Achieve has historically used a set of six criteria: rigor, coherence, focus, specificity, clarity/accessibility, and measurability. For the purposes of this analysis, the LSM were compared with the Common Core State Standards (CCSS) for Mathematics and analyzed with respect to these criteria.

Using a side-by-side perspective, looking at comparable grain sizes of the two sets of standards, it appears that they are very much alike. In fact, at the high school level there are only two standards intended for all students in the CCSS with no match in the high school LSM while there are *no* LSM standards that are not matched in the CCSS standards. In Grades K through 8 there were also very few differences and the LSM has added only *four* standards with no direct match in the CCSS. The LSM has modified some CCSS standards by splitting them into smaller parts, adding examples, or making slight changes to wording. In some cases the changes may be deemed as beneficial, but in others, changes have led to a loss of clarity. This report outlines these issues.

The key differences between the CCSS and the LSM are as follows:

- The CCSS includes the Standards for Mathematical Practice while the LSM has no similar counterpart.
- The LSM lacks a glossary and are therefore missing the definitions of key terms.
- Across all levels of the LSM cluster headings have been removed, eliminating an essential structural component.
- In addition to the removal of the cluster headings, at the high school level the LSM has also removed the organizational structure at the domain level.
- Modeling, as a conceptual category, has been removed along with all indications of standards being associated with it.
- There are no additional higher standards, such as the CCSS (+) standards, intended for students who plan to take advanced courses.

[1] http://www.louisianabelieves.com/docs/default-source/academic-standards/la-standards-for-math-1-25-16-draft.pdf?sfvrsn=6

- Finally, the draft of the LSM reviewed lacks an introductory narrative to orient the reader and, in particular, lacks grade level overviews of the standards. The LSM is essentially a less structured listing of the content standards in the CCSS.

Review of Louisiana's Draft Mathematics Standards Using Achieve's Criteria for the Evaluation of College- and Career-Ready Standards

This report provides a review of the draft of the Louisiana Standards for Mathematics (LSM) released in January 2016. This draft includes standards for each grade, from K through 8, along with standards for high school courses in Algebra I, Algebra II, and Geometry. Unlike many other sets of standards, the document reviewed includes no front matter. The LSM could be improved by including an introduction to the standards, as well as grade level introductions or summaries, and a glossary of terms. Also needed is an emphasis on practices or processes that all mathematics educators should be looking for and developing in their students. Including practice standards in the LSM, such as those in the CCSS Standards for Mathematical Practice, would further the cause of encouraging deeper mathematical thinking in Louisiana classrooms by both students and teachers.

For Grades K through 8, the standards are organized by grade. Each grade consists of sections, and each section consists of a single list of standards. The names of the sections and appearance at specific grade levels match that of the CCSS, as shown below:

	K	1	2	3	4	5	6	7	8
Counting and Cardinality	■								
Numbers and Operations in Base Ten	■	■	■	■	■	■			
Numbers and Operations - Fractions				■	■	■			
Ratios and Proportional Relationships							■	■	
The Number System							■	■	■
Operations and Algebraic Thinking	■	■	■	■	■	■			
Expressions and Equations							■	■	■
Functions									■
Measurement and Data	■	■	■	■	■	■			
Geometry	■	■	■	■	■	■	■	■	■
Statistics and Probability							■	■	■

While the structure of the CCSS includes clusters of standards and headers describing those clusters, there is no similar structure in the LSM. In some cases that missing structure has an effect on what is conveyed in a particular content standard. Examples are provided in this report in the section on Coherence.

For high school, the standards are organized first by course: Algebra I, Algebra II, and Geometry. Each course has sections and each section has a list of standards. This is in contrast to the CCSS where standards are organized by conceptual category, domain, and cluster. The sections in the LSM courses align to the conceptual categories in the CCSS, but at the high school level in the LSM there are no structures similar to the domains and clusters of the CCSS. In addition Modeling is not explicitly mentioned in the LSM high school course standards and, unlike the CCSS, the standards associated with modeling are not specifically highlighted. The LSM sections (or conceptual categories in the CCSS) are as follows:

- Number and Quantity
- Algebra
- Functions
- Geometry
- Statistics and Probability

One interesting result of the organization in the LSM is that there are only two sections in the Geometry course: *Geometry* and *Statistics and Probability*. The Geometry section of the Geometry course is simply a list of 36 standards without any sort of additional grouping, organization, or clarification. For implementation purposes it may be helpful to provide additional structure and organization to the standards. Possible implications of this are described in the Coherence section of this document.

To inform the analysis, Achieve generated side-by-side charts that provide full alignment and commentary of the CCSS as compared to the LSM from Kindergarten through Algebra I, Algebra II, and Geometry. The chart uses the CCSS as the organizing structure in the left column. Each LSM standard is used in the alignment chart at least once in the columns directly to the right of the CCSS column. Commentary on the alignment is in the column on the far right.

There is no abbreviated coding scheme in the LSM. To help map the two sets of standards onto each other the reviewers used a coding scheme similar to the CCSS. As such, in this document and the accompanying charts the following abbreviations to reference the sections of the LSM standards are used:

LSM K-8
Counting and Cardinality (C)
Numbers and Operations in Base Ten (NBT)
Numbers and Operations – Fractions (NF)
Ratios and Proportional Relationships (RP)
The Number System (NS)
Operations and Algebraic Thinking (OA)
Expressions and Equations (EE)
Functions (F)
Measurement and Data (MD)
Geometry (G)
Statistics and Probability (SP)
LSM HS
Number and Quantity (NQ)
Algebra (A)
Functions (F)
Geometry (G)
Statistics and Probability (SP)

For purposes of this analysis, for Grades K-8 the report indicates the standard by grade level, section, and list number. For example, 4.MD.3 refers to the third standard of the Grade 4 section *Measurement and Data*. For purposes of this analysis, the reviewers used A1 for Algebra I, A2 for Algebra II, and G for Geometry to distinguish the high school course associated with the standard. For example, A2.SP.3 refers to the course *Algebra II*, the section *Statistics and Probability*, and the third standard in the list.

Rigor

Rigor refers to the intellectual demand of the standards. It is the measure of how closely a set of standards represents the content and cognitive demand necessary for students to succeed in credit-bearing college courses without remediation and in entry-level, quality, high-growth jobs. Rigorous standards should reflect, with appropriate balance, conceptual understanding, procedural skill and fluency, and applications. For Achieve's purposes, the CCSS represent the appropriate threshold of rigor.

At the content standard level, considering the standards intended for all students, the LSM and the CCSS are very similar. However, the LSM modified a few of the CCSS in such a way as to impact rigor. In some cases the expectation is raised when compared to the corresponding CCSS. For instance, the two standards below illustrate the inclusion of *applying* as well as *explaining*:

CCSS	LSM (HS)	Comment
5.NBT.2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	5.NBT.2. Explain and apply patterns in the number of zeros of the product when multiplying a number by powers of 10. Explain and apply patterns in the values of the digits in the product or the quotient, when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. *For example, 10^0 = 1, 10^1 = 10 ... and 2.1 x 10^2 = 210.*	In this standard the LSM added "and apply" to the explain portion. This increases the demand for the LSM standard. Note: It is not clear which part of the standard the example is intended to exemplify.
G.CO.9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	G.G.9. Prove and apply theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	Prove became "Prove and apply." (This also happened in G.G.10 and G.G.11.)

In one case, the rigor of the standard was shifted from *derive* to *apply:*

CCSS	LSM Algebra II	Comment
A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*	A2.A.3. Apply the formula for the sum of a finite geometric series (when the common ratio is not 1) to solve problems. *For example, calculate mortgage payments.*	"Derive" and "use" became "apply." The CCSS version requires a higher depth of knowledge. In this LMS only the "use" part of the CCSS is required.

At the standard level there is little difference between the CCSS and the LSM. There are concerns, though, with the removal of the Standards for Mathematical Practice. There is wide agreement in the value and importance of including practice standards for mathematics. In addition to the CCSS states, Texas, Nebraska, Virginia, Indiana, South Carolina, Oklahoma, and Alaska all have some sort of practice or process standards in place. Louisiana students would benefit greatly from inclusion of these practices as part of their requirements.

Additionally, one component of rigor is in the application of the mathematics. One way this is accomplished in the CCSS is in the inclusion of the conceptual category of Modeling along with the identification of specific standards that require one or more aspects of mathematical modeling. Since the CCSS that are indicated as modeling standards are included, it is not that those opportunities are missing in the LSM, but rather that they are not clearly indicated. In omitting the modeling indicators, LSM risks missing key opportunities to highlight mathematical application.

The CCSS also includes more rigorous standards that go beyond the standards intended for all students. These standards, known as the (+) standards, are designed for students who plan to do further study in areas such as calculus, statistics, or discrete mathematics. There are no similar standards in the LSM.

Coherence

Coherence refers to how well a set of standards conveys a unified vision of the discipline, establishing connections among the major areas of study and showing a meaningful progression of content across the grades, grade spans, and courses.

The differences in coherence between the CCSS and the LSM are subtle. As with rigor, the two sets of standards look very much alike at the grade- and content-level comparison of standards. From that perspective, the coherence that is intended by the CCSS is not explicitly *contradicted* in the LSM. However a critical component of the coherence in the CCSS is in the structure that surrounds the content standards and that is missing in the LSM. As mentioned earlier, the CCSS structure also includes cluster headings and grade level introductions for the K through 8 standards and, at the high school level, there are domains and clusters within the conceptual categories. The loss of these structures in the LSM may result in a lack of coherence in communicating how the standards are connected both within and across the grade levels or courses. The following table offers a few examples:

CCSS Cluster	CCSS Standard(s)	Comment

2.OA. Work with equal groups of objects to gain foundations for multiplication.	2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. 2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	In the CCSS the intention of these standards is to build foundations for multiplication. Without the cluster header this coherence may be lost. This intention is not explicit in the LSM though these standards have identical matches in the LSM.
3.NF. Develop understanding of fractions as numbers.	3.NF.1. Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b. 3.NF.2. Understand a fraction as a number on the number line; represent fractions on a number line diagram. 3.NF.2a Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. 3.NF.2b Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. 3.NF.3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. 3.NF.3a Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. 3.NF.3b Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. 3.NF.3c Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram. 3.NF.3d Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >,	In the CCSS the intention is clear that *all* of these standards are serving to develop fractions as numbers. That is, a single fraction is a single number. These standards have essentially identical matches in the LSM. (See the accompanying chart.) However, in the LSM this intention is not explicit and the handling of fractions could become fractured.

	=, or <, and justify the conclusions, e.g., by using a visual fraction model.	
3.MD. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	3.MD.8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	Without the cluster heading or a glossary the notion that the perimeter is an attribute (rather than a measurement) is missing from the LSM. This standard has an identical match in the LSM.
6.NS. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?	The CCSS cluster heading serves to remind that multiplication and division of fractions should build from earlier understandings of multiplication and division. This is often lost in practice. This intention is not explicit in the LSM although this standard has an identical match in the LSM.
7.EE. Use properties of operations to generate equivalent expressions.	7.EE.1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. 7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."	In this case the CCSS cluster heading indicates that these standards are all about working with equivalent expressions. It is possible that these standards could be interpreted without this in mind. These standards have nearly identical matches in the LSM. (See the accompanying chart.)
8.EE. Understand the connections between proportional relationships, lines, and linear equations.	8.EE.5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* 8.EE.6. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y* = *mx* for a line through the origin and the	The reason for these standards in the CCSS is to make connections between proportional relationships, lines, and linear equations. That intention is not evident in the LSM though these standards have identical matches in the LSM.

	equation $y = mx + b$ for a line intercepting the vertical axis at b.	

At the high school level the CCSS are organized at the highest level by conceptual category, then by domain, then cluster, and finally the standard. In the LSM the standards are organized by course, conceptual category, and the standard. To see the impact of the difference consider how a given standard might look in each case:

CCSS Conceptual Category	CCSS Domain	CCSS Cluster	CCSS Standard
Geometry	Similarity, Right Triangles, and Trigonometry (SRT)	A. Prove theorems involving similarity	4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
LSM Course	**LSM Section**	**LSM Standard**	
Geometry	Geometry	17. Prove and apply theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity; SAS similarity criteria; SSS similarity criteria; ASA similarity.*	

The CCSS reader sees that this standard is grouped with other standards on similarity, and, in particular, proving theorems involving similarity. In the LSM there is no similar support. Interestingly, there is another “Prove theorems about triangles” standard in the LSM, but the list of theorems is different:

CCSS Conceptual Category	CCSS Domain	CCSS Cluster	CCSS Standard
Geometry	Congruence (CO)	C. Prove geometric theorems	10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
LSM Course	**LSM Section**	**LSM Standard**	
Geometry	Geometry	10. Prove and apply theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	

The distinction between these two is very clear in the CCSS. One deals with theorems of similarity. The other deals with theorems of congruence. There is no similar level of clarity in the LSM. If stated without the examples, these are simply two identical standards in a list of 34 others. An additional example shows how the LSM may lose the connection to using coordinates in the proof of the criteria for parallel and perpendicular slopes:

<table>
<tr><th>CCSS Conceptual Category</th><th>CCSS Domain</th><th>CCSS Cluster</th><th>CCSS Standard</th></tr>
<tr><td>Geometry</td><td>Expressing Geometric Properties with Equations (GPE)</td><td>B. Use coordinates to prove simple geometric theorems algebraically</td><td>5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).</td></tr>
<tr><th>LSM Course</th><th>LSM Section</th><th colspan="2">LSM Standard</th></tr>
<tr><td>Geometry</td><td>Geometry</td><td colspan="2">28. Prove the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).</td></tr>
</table>

The LSM counterpart to CCSS, G.GPE.5, does not reference the use of coordinates in the proof and that aspect of this standard's requirement may be lost in Louisiana classrooms.

In addition to the issues of coherence around the structure of the standards, there are a few instances where coherence in the progression of content was lost by modifying a standard. In one case, the modification breaks the coherence of the fractions progression by requiring multiplication of fractions by fractions one year before the introduction of fraction multiplication.

CCSS	LSM	Comment
4.MD.2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	4.MD.2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving whole numbers and/or simple fractions (addition and subtraction of fractions with like denominators and multiplying a fraction times a fraction or a whole number), and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that	Fractions multiplied by fractions are now included, even though it is a Grade 5 topic.

	feature a measurement scale.	

In one instance, the LSM for Algebra I requires that students be able to reference an Algebra II concept:

CCSS	LSM Algebra I	LSM Algebra II	Comment
A.REI.11. Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★	A1.A.16. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, piecewise linear (to include absolute value), and exponential functions.	A2.A.14. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	LSM A1 includes rational functions, but rational expressions and equations are otherwise only addressed in Algebra II (See A2.A.7, A2.A.8, and A2.A.10).

Focus

High-quality standards establish priorities about the concepts and skills that should be acquired by students. A sharpened focus helps ensure that the knowledge and skills students are expected to learn are important and manageable in any given grade or course.

While the CCSS and the LSM are extremely similar at the standard level, there are a few instances of a shift in focus. Relating area to multiplication and addition and recognition of area as additive is found in Grade 3 in the CCSS, but has been moved to Grade 4 in the LSM (4.MD.8), even though it connects well to LSM 3.MD.7c in Grade 3. Mean Absolute Deviation is found in Grade 6 in the CCSS, but has been moved to Grade 7 in the LSM (7.SP.3). Solving linear inequalities has been added to Grade 6 (6.EE.7).

There also have been a few standards added to the LSM that are not addressed in the CCSS. For example there are additional LSM standards involving an understanding of currency in Grades K, 1, and 3. (K.MD.4, 1.MD.5, and 3.MD.9). There is also a new standard in Grade 5 (5.NF.4b) that seems to result from a split in a CCSS standard[2].

For high school, the LSM added the clarification in A2.A.12 that students will be expected to solve systems of three equations and three unknowns. Overall, the LSM high school standards match all but two CCSS standards intended for all students:

- G.GPE.2 Derive the equation of a parabola given a focus and directrix.
- S.ID.1. Represent data with plots on the real number line (dot plots, histograms, and box plots).

Given that the content in LSM S.ID.1 largely overlaps with LSM 6.SP.4, the content intended by both sets of standards nearly identical. The high school standards would greatly benefit from clarification as to the limits of certain topics that overlap in Algebra I and Algebra II. For example, A1.A.16 includes rational functions. Are rational functions intended in Algebra I? Clear limits of expectations should be articulated with respect to linear, quadratic, exponential, and absolute value functions. Additionally, there are many instances where a standard in Algebra I is identical to a standard in Algebra II:

LSM Algebra I	LSM Algebra II	Comment
A1.N.3. Define appropriate quantities for the purpose of descriptive modeling.	A2.N.3. Define appropriate quantities for the purpose of descriptive modeling.	The progression of expectation from Algebra I to Algebra II is not clear.

The same issue exists for the following:

LSM Algebra I	**LSM Algebra II**
A1.A.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.	A2.A.2. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
A1.F.7. Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	A2.F.3. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
A1.F.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	A2.F.4. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
A1.F.10a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	A2.F.6a. Determine an explicit expression, a recursive process, or steps for calculation from a context. "
A1.F.15. Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.	A2.F.12. Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.
A1.SP.4. Represent data on two quantitative variables on a scatter plot, and describe how the	A2.SP.2. Represent data on two quantitative variables on a scatter plot, and describe how the variables are

[2] This modification is addressed in the Clarity section.

variables are related.	related.

In one case a small change in wording changed the focus of the standard:

CCSS	LSM (K-8)	Comment
7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	7.RP.3. Use proportional relationships to solve multistep ratio and percent problems of simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error.	Replacing “Examples:” with "of" makes the CCSS examples part of the standard and appears to limit the LSM counterpart to those specific types of tasks.

Additionally, the inclusion of an example (perhaps inadvertently) served to lower the expectation of one standard:

CCSS	LSM (K-8)	Comment
4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. *Example: Twenty-five people are going to the movies. Four people fit in each car. How many cars are needed to get all 25 people to the theater at the same time?*	The LSM added an example to this standard that may fall short of the overall intention of the standard. There is a danger in that this becomes a prototype of "multistep" when this could aim much deeper. (See https://www.illustrativemathematics.org/content-standards/4/OA/A/3/tasks/1289, for example.)

Specificity

Quality standards are precise and provide sufficient detail to convey the level of performance expected without being overly prescriptive. Those that maintain a relatively consistent level of precision are easier to understand and use. Those that are overly broad or vague leave too much open to interpretation, while atomistic standards encourage a checklist approach to teaching and learning.

Given the similarities of the CCSS and LSM at the standard level, the specificity in the two sets of standards is very close to the same. Some changes, however, may have resulted in too much specificity and may warrant review:

CCSS	LSM (K-8)	Comment
8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers, show that the decimal expansion repeats eventually. Convert a decimal expansion that repeats eventually into a rational number by analyzing repeating patterns.	The addition of "analyzing repeating patterns" seems to unnecessarily restrict conversion methods.
8.G.6. Explain a proof of the Pythagorean Theorem and its converse.	8.G.6. Explain a proof of the Pythagorean Theorem and its converse using the area of squares.	The LSM requires the proof to be based on the area of squares. There are many proofs of the Pythagorean theorem so it is unclear why this modification was made.

As another example of the need for cluster headings, in a couple of cases the removal of the cluster headings accompanied a significant change in specificity for two standards in Grade 8:

CCSS	LSM (K-8)	Comment
8.G.2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	8.G.2. Explain that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Rotations are only about the origin and reflections are only over the y-axis and x-axis in Grade 8.)	The added notes in these two LSM standards unnecessarily limit the transformation work with figures to the coordinate plane. This is not the case in the CCSS, as is clear from the cluster heading, which indicates the use of physical models, transparencies, or geometry software. This limitation restricts the standard and will also restrict the adoption of materials that have been designed to align to the CCSS.
8.G.4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two- dimensional figures, describe a sequence that exhibits the	8.G.4. Explain that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two- dimensional figures, describe a sequence that exhibits the similarity between them. (Rotations are only about the origin, dilations only use the origin	

similarity between them.	as the center of dilation, and reflections are only over the *y*-axis and *x*-axis in Grade 8.)	

Clarity/Accessibility

High-quality standards are clearly written and presented in an error-free, legible, easy-to-use format that is accessible to the general public.

Overall, when compared at the standard level, the LSM are generally clear. The lack of surrounding structure leads to issues of overall clarity and accessibility. The lack of domains and clusters arguably makes the standards more difficult to fully grasp. It is important that the connections within and between grade levels are clear to users, but, unlike the CCSS, many of those connections are left to the user in the LSM.

Sometimes LSM divided the standards into smaller parts. Splitting a standard should always be done with care. While it may seem to make things clearer, splitting can also contribute to a separation of connected ideas and viewing standards as a checklist. In this case the modified language could be clearer:

CCSS	LSM	Comment
KCC.5. Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.	K.C.5. Count to answer "How many?" questions K.C.5a. Count objects up to 20, arranged in a line, a rectangular array, or a circle. K.C.5b. Count objects up to 10 in a scattered configuration. K.C.5c. When given a number from 1-20, count out that many objects.	The LSM splits this CCSS, but the content is essentially the same. This split into such small grain sized parts may encourage the checklist approach to addressing the standards. Also, it would be clearer to say, for example, "count up to 20 objects" rather than “count objects up to 20.”

In one case the standard was split and information from the missing cluster header was also added to the standard:

CCSS	LSM	Comment
K.NBT.1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand	K.NBT.1. Gain understanding of place value. K.NBT.1a. Understand that the numbers 11–19 are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. K.NBT.1b. Compose and decompose numbers 11 to 19 using place value (e.g., by using objects or drawings). K.NBT.1c. Record each composition	The LSM splits this CCSS, but the content is essentially the same. K.NBT.1 clarifies what is missing from the CCSS cluster heading that was removed, "Work with numbers 11-19 to gain foundations for place value."

that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	or decomposition using a drawing or equation (e.g., 18 is one ten and eight ones, 18 = 1 ten + 8 ones, 18 = 10 + 8).	

While LSM places earlier emphasis on money, there is a need for further clarification in some of the added standards:

CCSS	LSM	Comment
N/A	1.MD.5. Determine the value of a collection of coins up to $.50. (Pennies, nickels, dimes, and quarters in isolation; not to include a combination of different coins.)	Will the units be in dollars or in cents? The notation in the LSM seems to intend that dollars be the unit.
N/A	3.MD.9. Solve word problems involving pennies, nickels, dimes, quarters, and bills greater than one dollar, using the dollar and cent symbols appropriately.	This standard seems to inadvertently exclude one-dollar bills.

In one case what was perhaps intended to improve clarity may have reduced *mathematical* clarity:

CCSS	LSM	Comment
4.NBT.1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*	4.NBT.1. Recognize that in a multi-digit whole number less than or equal to 1,000,000, a digit in one place represents ten times what it represents in the place to its right. *Examples: (1) recognize that 700 ÷ 70 = 10; (2) in the number 7,246, the 2 represents 200, but in the number 7,426 the 2 represents 20, recognizing that 200 is ten times as large as 20, by applying concepts of place value and division.*	The intention is to limit the tasks to numbers equal to or less than 1,000,000. The statement, however, reads as though this may be true *only* for such numbers.

In one case the addition of an example ended up blurring the distinction between two standards:

CCSS	LSM	Comment
4.NF.3a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.	4.NF.3a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole (*Example: 3/4 = 1/4 + 1/4 + 1/4*).	This new example blurs the distinction between this and the following standard (4.NF.3b) as the same sort of example is provided in both standards.

4.NF.3b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.	4.NF.3b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*	

A number of other small modifications should be re-evaluated for clarity:

CCSS	LSM	Comment
7.EE.1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	7.EE.1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients to include multiple grouping symbols (e.g., parentheses, brackets, and braces).	Braces are removed from Grade 5 and are now found in Grade 7. This standard explicitly calls for *multiple* grouping symbols now. This could mean [3+x](2-x) or it could be about nested groups. This distinction should be made clearer.
7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	7.G.2. Draw (freehand, with ruler and protractor, or with technology) geometric shapes with given conditions. (Focus is on <u>triangles from three measures of angles or sides</u>, noticing when the conditions determine one and only one triangle, more than one triangle, or no triangle.)	The underlined part of the sentence is the parenthesis is unclear. Perhaps it should read, "The focus is on drawing triangles from..."
F.IF.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.�	A1.F.6. Calculate and interpret the average rate of change of a linear, quadratic, piecewise linear (to include absolute value), and exponential function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	The position of "presented symbolically" makes it seem that this applies only to exponential functions.

G.SRT.4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	G.G.17. Prove and apply theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity; SAS similarity criteria; SSS similarity criteria; ASA similarity.*	This addition to the CCSS might be misunderstood. Is the intention to prove or just apply SAS, SSS, and ASA? And as these are listed singularly, they should be each referred to as criterion.
1.G.2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.4	1.G.2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) and three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.	By switching from “and” to “or” this implies composing two-dimensional and three-dimensional shapes with each other.
5.NF.4a Interpret the product (*a*/*b*) x *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* x *q* ÷ *b*. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*	5.NF.4a. Interpret the product (*m*/*n*) x *q* as m parts of a partition of q into n equal parts; equivalently, as the result of a sequence of operations, *m* x *q* ÷ *n*. *For example, use a visual fraction model to show understanding, and create a story context for (m/n) x q.* 5.NF.4b. Construct a model to develop understanding of the concept of multiplying two fractions and create a story context for the equation. [In general, (*m*/*n*) x (*c*/*d*) = (*mc*)/(*nd*)]	LSM 5.NF.4b seems to be a split of CCSS 5.NF.4.a. However it is not clear what is intended by, "Construct a model to develop understanding of the concept…” The use of the term, “model” may be confusing for teachers who struggle to understand what mathematical modeling is. Is the student to model a situation using mathematics or is this about creating a visual model for some mathematics? Does the model they provide have to actually develop understanding or can it just show understanding? Also the standard mentions "the equation" but no equation is indicated.
7.G.6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles,	7.G.6. Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (Pyramids limited to surface area	The only three-dimensional objects addressed in the standard are cubes and right prisms. The parenthetical about pyramids does not clarify the standard.

quadrilaterals, polygons, cubes, and right prisms.	only.)	

Measurability

Standards should focus on results rather than the processes of teaching and learning. They should make use of performance verbs that call for students to demonstrate knowledge and skills, with each standard being measurable, observable, or verifiable in some way.

The LSM generally reflect a comparable level of measurability to that of the CCSS. As mentioned earlier, though, due to the placement of standards in courses, the expectations of the high school standards, particularly between Algebra I and Algebra II, need to be made clearer.

Summary

From a standard-by-standard perspective, the LSM is nearly identical to the CCSS. The expectations contained within the LSM can be used to prepare students for postsecondary education and careers. It is also likely that educators in Louisiana could adapt instructional materials for Grades K through 8, including professional development and assessments, created for the state's previously adopted standards. For Algebra I, Algebra II and Geometry, while the standards maintain a very strong match, care will need to be taken to ensure that adopted materials and assessments align to the Louisiana allocation of standards course-by-course[3].

As indicated in this report, there are a number of issues that remain to be addressed. Overall, though, the draft standards would immediately benefit from the addition of supporting materials, practices, and a coding scheme. Achieve strongly recommend reconsidering the removal of the additional content structures (domains and clusters) that are found in the CCSS.

[3] There is no formal allocation of standards to high school courses in the CCSS, so this is true for all CCSS states as well.

Appendix: The Criteria Used for the Evaluation of College- and Career-Ready Standards in English Language Arts and Mathematics

Criteria	Description
Rigor: What is the intellectual demand of the standards?	Rigor is the quintessential hallmark of exemplary standards. It is the measure of how closely a set of standards represents the content and cognitive demand necessary for students to succeed in credit-bearing college courses without remediation and in entry-level, quality, high-growth jobs. For Achieve's purposes, the Common Core State Standards represent the appropriate threshold of rigor.
Coherence: Do the standards convey a unified vision of the discipline, do they establish connections among the major areas of study, and do they show a meaningful progression of content across the grades?	The way in which a state's college- and career-ready standards are categorized and broken out into supporting strands should reflect a coherent structure of the discipline and/or reveal significant relationships among the strands and how the study of one complements the study of another. If college- and career-ready standards suggest a progression, that progression should be meaningful and appropriate across the grades or grade spans.
Focus: Have choices been made about what is most important for students to learn, and is the amount of content manageable?	High-quality standards establish priorities about the concepts and skills that should be acquired by graduation from high school. Choices should be based on the knowledge and skills essential for students to succeed in postsecondary education and the world of work. For example, in mathematics, choices should exhibit an appropriate balance of conceptual understanding, procedural knowledge and problem solving skills, with an emphasis on application. In English language arts, standards should reflect an appropriate balance between literature and other important areas, such as informational text, oral communication, logic, and research. A sharpened focus also helps ensure that the cumulative knowledge and skills that students are expected to learn is manageable.
Specificity: Are the standards specific enough to convey the level of performance expected of students?	Quality standards are precise and provide sufficient detail to convey the level of performance expected without being overly prescriptive. Standards that maintain a relatively consistent level of precision ("grain size") are easier to understand and use. Those that are overly broad or vague leave too much open to interpretation, increasing the likelihood that students will be held to different levels of performance, while atomistic standards encourage a checklist approach to teaching and learning that undermines students' overall understanding of the discipline. Also, standards that contain multiple expectations may be hard to translate into specific performances.
Clarity/Accessibility: Are the standards clearly written and presented in an error-free, legible, easy-to-use format that is accessible to the general public?	Clarity requires more than just plain and jargon-free prose that is also free of errors. College- and career-ready standards also must be communicated in language that can gain widespread acceptance not only from postsecondary faculty but also from employers, teachers, parents, school boards, legislators, and others who have a stake in schooling. A straightforward, functional format facilitates user access.
Measurability: Is each standard measurable, observable, or verifiable in some way?	In general, standards should focus on the results, rather than the processes of teaching and learning. College and career-ready standards should make use of performance verbs that call for students to demonstrate knowledge and skills and should avoid using those that refer to learning activities — such as "examine," "investigate," and "explore" — or to cognitive processes, such as "appreciate."