How Many Small Boxes?

Resource ID#: 48955

Primary Type: Lesson Plan

In this lesson students will extend their knowledge of volume from using whole numbers to using fractional units. Students will work with adding, multiplying, and dividing fractions to find the volume of right rectangular prisms, as well as, determining the number of fractional unit cubes in a rectangular prism.
Subject(s): Mathematics
Grade Level(s): 6
Intended Audience: Educators E
Suggested Technology: Document Camera, Overhead Projector

Instructional Time: 1 Hour(s) 30 Minute(s)

Freely Available: Yes
Keywords: volume, rectangular prism, fraction
Instructional Component Type(s): Lesson Plan , Worksheet, Assessment , Formative Assessment
Instructional Design Framework(s): Direct Instruction , Confirmation Inquiry (Level 1) , Cooperative Learning

Resource Collection: CPALMS Lesson Plan Development Initiative

ATTACHMENTS

How many small boxes answer key.pdf
How many small boxes worksheet.pdf
We Pack and Ship Part A.pdf
We pack and ship part A answers.pdf
We pack and ship Part B.pdf
We Pack and Ship Part B answers.pdf
We Pack and Ship Part C.pdf
We Pack and Ship Part C answers.pdf

LESSON CONTENT

- **Lesson Plan Template:** Confirmatory or Structured Inquiry
- **Formative Assessment**

  The teacher will gather information during and after each inquiry activity.

  Part A:

  Assesses prior knowledge of finding volume by packing a prism with whole number unit cubes and also applying the volume formula V = lwh.

  Through observation and questioning during and after the activity, the teacher will assess students' understanding of the concept of volume using whole numbers.

  Part B:

  Students extend the skills from part A to include fractional units.

  Through observation and questioning during and after the activity, the teacher will assess and guide students' understanding of the concept that the same procedures or strategies used in Part A can be used for any type of number (in this case fractions).

  Part C:

  Students will go from the concrete to the abstract to determine the number of fractional unit cubes necessary to fill a given prism and to find the volume using the volume of the fractional unit cube and the volume formula V = lwh.

Through observation and questioning during and after the activity, the teacher will assess and guide students' understanding of the concept of decomposing the side lengths of the prism into fractional units (i.e. a side length of 3-1/2 to 10 halves) to determine the number of fractional unit cubes needed to fill the box. The teacher will also observe and assess students' understanding of the use of the formula V = lwh using fractions.

- **Feedback to Students**

  The student will receive feedback from the teacher and their peers during and after each part of the the inquiry activity through questions, discussions, and observations.

  Part A:

  Through feedback students should understand:

  - The length, width, and height of the prism is determined by the adding the number of unit cubes (i.e. 4 cubes long is 4 units)
  - To find the volume you can find the volume of one unit cube multiplied by the number of unit cubes
  - To find the volume you can use the volume formula V = lwh
  - When finding the dimensions of a rectangular prism the lwh can be interchanged (i.e. 2 x 3 x 4 is the same as 3 x 4 x 2)
  - The volume is determined by the number of unit cubes (i.e. you can change the dimensions, but as long as you have the same number of unit cubes the volume will not change)

  Part B:

  Through feedback students will understand:

  - Using fractions instead of whole numbers does not change the procedures and/or strategies used in Part A

  Part C:

  Through feedback students will understand:

  - Decomposing the side lengths of a prism into the length of the given unit will give the number of units cubes long, wide, and high (i.e. a length of 4-1/4 units with a unit cube of 1/4 is 17 unit cubes long).
  - The concepts from Parts A and B can be applied to Part C.
- **Summative Assessment**
  The attached assessment "How Many Small Boxes" will be used to check student understanding.
- **Learning Objectives: What will students know and be able to do as a result of this lesson?**

- Students will calculate the number of fractional unit cubes needed to fill a right rectangular prism.
- Students will calculate the volume of a right rectangular prism using the volume of a fractional unit cube multiplied by the number of unit cubes.
- Students will use the formula V = lwh to find the volume of a right rectangular prism with fractional side lengths.
- Students will understand volume is determined by the number of cubic units (unit cubes) contained in a rectangular prism.

- **Guiding Questions: What are the guiding questions for this lesson?**
  - What does volume mean?
  - How can you determine the number of unit cubes needed to fill a rectangular prism?
  - How can you use the volume of a unit cube to find the volume of a larger rectangular prism?
  - How does the volume change when the size of the unit cube changes?
  - Does the volume change when you change the dimensions of the prism but not the total number of unit cubes?
- **Prior Knowledge: What prior knowledge should students have for this lesson?**
  - Decompose a fraction into a sum of fractions (MACC.4.NF.2.3.b)
  - Add, multiply, and divide fractions (MCC.5.NF.1 and MACC.5.NF.2)
  - Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and by applying the formulasV = *l* x *w* x *h* and V = b x *h* (MCC.5.MD.3)
- **Introduction: How will the teacher introduce the lesson to the students?**

  Use a box of sugar cubes or other objects that are packed without gaps.

  - Hold up the sugar cube and the sugar cube box.
  - Discuss how things are packaged, like the sugar cubes, inside a box with no gaps. (There is no extra room between the cubes and sides of the box)
  - List other things that might be packaged the same way. (Anything that is shipped in quantities is usually packaged without gaps.)
  - Discuss why things are packaged this way. (To reduce cost, so they won't break, presentation - like the Tobbler boxes all fit together, etc.)
  - Ask, "How would you determine the size (dimensions) of the box needed?" (It would have to contain all the smaller packages.)
- **Investigate: What question(s) will students be investigating? What process will students follow to collect information that can be used to answer the question(s)?**

  Part A: Building boxes using whole number unit cubes

  Question: How do you determine the dimensions of the box needed to package 24 smaller boxes?

  - Group students in pairs.
  - Provide each pair of students with 24 cubes

- Provide each pair of students with a copy of the "We Pack and Ship" Activity Part A. For the teacher: Part A Answer Key.
- Have students complete the activity. (Note: The activity asks students to find the volume of the package. Students may need a short review on how to find the volume. If necessary, remind students of the formula: V = lwh).
- As students work on the activity, circulate to observe the strategies being used to determine the dimensions of the large box, as well as, their understanding of the task (see formative assessment and feedback to students section). Suggested guiding questions:
    - What strategies are you using to make your boxes? (Trial and error, start with 1 unit by 1 unit by 24, then increase the length by 1, etc.)
    - How did you find the length of one side? (counted the number of cubes)
    - How did you find the volume? (multiplied length x width x height; counted the number of cubes)
    - Can you find the volume if you know the volume of the small box? Explain. (Yes, but you would have to know the number of small boxes. The volume is the number of small boxes times the volume of the small box.)
    - Is the box with dimensions 2 units x 3 units x 4 units different from the box with dimensions 3 units x 2 units x 4 units? In what way? (What is labeled as the length, width, and height is different, but it doesn't change the actual box because you could turn it. The sides would still have the same dimensions.)
    - Did the volume change when the dimensions changed? Why not? (No, because the number and size of the small boxes didn't change they were just rearranged.)
- After students have completed the activity, have students share their box dimensions and discuss how they found each set of dimensions. Use the questions above to guide the discussion.
- Ask: What would happen if the dimensions of the small box changed?

Part B

Question: What would happen if the dimensions of the small box changed?

- Provide each pair of students with a copy of "We Pack and Ship" Part B. For the teacher: Part B Answer Key.
- Tell students to do all calculations using the fractions -- don't change anything to a decimal
- As students work on the activity, circulate and observe the strategies being used, as well as, their understanding of the task. (See formative assessment and feedback to students sections). Students should realize the same procedures used in Part A may be used in Part B. Suggested guiding questions:

    - How did you find the length of one side? (multiplied the length of one small box (unit cube) -- 1/2 unit -- by the number of cubes, or added 1/2 for each cube)
    - How did you find the volume of the small box? (be sure students multiplied the fractions correctly)
    - Can you use the volume of the small box to find the volume of the large box? How? (multiply the volume of the small box by the number of small boxes -- same as in part A)
    - How did the volume of the large box change when the dimensions of the small box changed? (The volume is less, because the volume of the small box is less. An extension to this question could be: by how much did the volume change? 3/24 or 1/8 the size or 8 times smaller)
- After students have completed the activity, have students share their large box dimensions. Facilitate a discussion on the strategies/procedures used to find the dimensions and volume. By the end of the discussion, students should understand the same procedures for finding the volume and determining the number of cubes is the same no matter what the dimensions of the unit cube are.
- Ask: Using the ideas from what we just finished, how can we find how many small boxes will fit in a large box?

Part C

Question: How can you determine the maximum number of small boxes that will fit into a large box?

- Provide each pair of students with a copy of "We Pack and Ship" Part C. For the teacher: Part C Answer Key.
- Tell students to do the calculations using fractions -- do not change to decimals
- As students work on the activity, circulate and observe the strategies being used, as well as, their understanding of the task. (See formative assessment and feedback to students section). Suggested guiding questions:
    - How can you determine how many small boxes will fit across the length of the large box? (the length of the large box divided by the length of the small box)
    - How can you determine how many small boxes will fit in the bottom layer (base) of the large box? (find how many will fit in the length and how many fit in the width and multiply; or find the area of the base of the large box and divide it by the area of the base of the small box)
    - If you know how many small boxes will fit in the bottom layer (base) of the box, can you find how many will fit in the whole box? How? (I would need to know how many boxes high would fit in the large box and multiply that number by the number of boxes in the bottom layer.)
    - In the formula V = bh, what does b stand for? How do you calculate it?
- After students have completed the activity, facilitate a discussion on the strategies/procedures used to find the number of small boxes, as well as, how to find the volume of the large box. Students should understand to find the number

of small boxes, they must determine (through decomposition or division) the number of boxes that will fit into the length, width, and height.

- Ask students to summarize (as a whole group -- this will be assessed again in the summative assessment)
- How can you find the number of unit cubes (small boxes) in a rectangular prism (large box)? (by finding how many cubes will fit in the length, width, and height and multiplying)
- What are 3 different ways to find the volume of a rectangular prism? (Using the formula V = lwh, Using the formula V = bh, and Using the volume of the unit cube and multiplying it by the number of unit cubes in the rectangular prism)
- Have each student independently complete the summative assessment "How Many Small Boxes?"

How many small boxes answer key.

- **Analyze: How will students organize and interpret the data collected during the investigation?**
  Students will organize data in the tables provided on the activity worksheets. They will analyze and interpret the data through questions asked during the investigation.
- **Closure: What will the teacher do to bring the lesson to a close? How will the students make sense of the investigation?**
  Students will summarize how to find the number of unit cubes in a rectangular prism and identify 3 different ways to find the volume of a rectangular prism.

  Students will complete independently the summative assessment "How Many Small Boxes?"

ACCOMMODATIONS & RECOMMENDATIONS

- **Accommodations:**
  - To accommodate struggling students, provide fractional unit cubes with the unit written on it. This will help students to visually see what to add together for the length, width, and height.
  - Provide additional practice in adding, multiplying, and dividing fractions.
  - For the abstract (Part C) provide manipulatives for the student to "build" the given box.
  - Provide translations, dictionaries, and/or examples of unfamiliar vocabulary words for English Language Learners.
- **Extensions:** For advanced students, analyze change in dimensions. How many times greater is the volume when you change one, two, or three of the dimensions and why. Is there a pattern? Can you find a rule for change in dimension?
- **Suggested Technology:** Document Camera, Overhead Projector

- **Special Materials Needed:** Enough unit cubes for each pair of students to have at least 24.

Additional Information/Instructions

By Author/Submitter

This resource is likely to support student engagement in the following the Mathematical Practices:

MAFS.K12.MP.2.1 Reason abstractly and quantitatively as students shift from building and counting the number of cubes in a rectangular prism to using the formula for to find the volume

MAFS.K12.MP.7.1 Look for and make use of structure as students note the pattern and extend their knowledge in the use of the dimensions (whole numbers or fractions) to find the volume

MAFS.K12.MP.8.1 Look for and express regularity in repeated reasoning as students connect earlier knowledge of V=*lwh* using whole numbers to using fractions.

Some of the song, *Little Boxes* from http://www.youtube.com/watch?v=2_2lGkEU4Xs , could be played before the start of the lesson.

SOURCE AND ACCESS INFORMATION

Contributed by: Erin OBrien
Name of Author/Source: Erin OBrien,
District/Organization of Contributor(s): Brevard
Is this Resource freely Available? Yes
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Related Standards

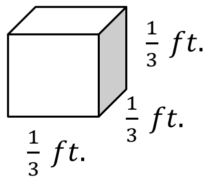
Name	Description
MAFS.6.G.1.2:	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = B\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Name: ___________________________________ Date: ____________________ Period: ____

How many small boxes?

On all problems, use a separate sheet of paper to neatly show your work.

Part A: Jody makes sculptures. Each sculpture is packaged in a square box pictured below.



Jody ships her sculptures in boxes with a length of $2\frac{1}{3}\ feet$, width $2\ feet$, and height $1\frac{2}{3}\ feet$.

What is the maximum number of sculptures she can ship in one box? ________

Part B: Show how to find the volume of the shipping box in three ways:

1) Using the volume of the sculpture box

2) Using the formula V = lwh

3) Using the formula V = Bh

Part C: Jody has to ship a special order of 36 sculptures. What are the dimensions of two possible shipping box sizes that will fit all 36 sculptures without extra space.

Box 1: _______ x ________ x _______

Box 2: _______ x ________ x _______

HOW MANY SMALL BOXES ANSWER KEY

Part A: What is the maximum number of sculptures she can ship in one box?

Length: $\frac{7}{3} \div \frac{1}{3} = 7$ Width: $\frac{2}{1} \div \frac{1}{3} = 6$ Height: $\frac{5}{3} \div \frac{1}{3} = 5$

7 x 6 x 5 = 210 sculptures

Part B: Show how to find the volume of the shipping box in three ways:

1) Using the volume of the sculpture box

Volume = number of sculpture boxes x volume of sculpture box

Volume of sculpture box = $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$

Volume = $210 \times \frac{1}{27} = \frac{210}{27} = 7\frac{7}{9}\ cubic\ feet$

2) Using the formula V = lwh

$$V = 2\frac{1}{3} \times 2 \times 1\frac{2}{3}$$

$$V = \frac{7}{3} \times \frac{2}{1} \times \frac{5}{3} = \frac{70}{9} = 7\frac{7}{9}\ cubic\ feet$$

3) Using the formula V = Bh

$$B = 2\frac{1}{3} \times 2 = \frac{7}{3} \times \frac{2}{1} = \frac{14}{3}$$

$$V = \frac{14}{3} \times 1\frac{2}{3} = \frac{14}{3} \times \frac{5}{3} = \frac{70}{9} = 7\frac{7}{9}\ cubic\ feet$$

Part C: Jody has to ship a special order of 36 sculptures. What are the dimensions of two possible shipping box sizes that will fit all 36 sculptures without extra space.

The volume is $1\frac{1}{3}$ cubic feet. Any combination that allows for 36 - $\frac{1}{3}$ cubes will work.

Possible answers in feet:

Box 1: $\frac{1}{3} \times 1\frac{1}{3} \times 3$

Box 2: $\frac{2}{3} \times \frac{2}{3} \times 3$

We Pack and Ship

Part A

You work at We Pack and Ship. A customer brings you 24 of the same size boxes to be packaged in one large box. The boxes must be packed with no gaps or extra room. Each box is 1 unit long, 1 unit wide, and 1 unit high. Complete the chart below, to find the possible dimensions and volume of the large box.

(use the blocks to help you determine the dimensions of the large box)

Length **in units**	Width **in units**	Height **in units**	Volume **in cubic units**

1. What is the volume of the small box? ________
2. How does the volume of the small box compare to the volume of each large box?

   ____

3. How can you find the volume of the large box using the volume of the small box?

   ____

We Pack and Ship

Part A

You work at We Pack and Ship. A customer brings you 24 of the same size boxes to be packaged in one large box. The boxes must be packed with no gaps or extra room. Each box is 1 unit long, 1 unit wide, and 1 unit high. Complete the chart below, to find the possible dimensions and volume of the large box.

(use the blocks to help you determine the dimensions of the large box)

Length **in units**	Width **in units**	Height **in units**	Volume **in cubic units**
1 unit	1 unit	24 units	24 cubic units
2 units	1 unit	12 unit	24 cubic units
3 units	1 unit	8 units	24 cubic units
4 units	2 units	3 units	24 cubic units
6 units	2 units	2 units	24 cubic units

1. What is the volume of the small box? 1 cubic unit
2. How does the volume of each of the large boxes compare to the volume of small box?

   The volume of the large box is 24 times larger than the volume of the small box.
3. How can you find the volume of the large box using the volume of the small box?

   Multiply the volume of the small box times the number of small boxes.

We Pack and Ship

Part B:

Another customer brings you 24 of the same size boxes to be packaged in one large box. The boxes must be packed with no gaps or extra room. Each box is $\frac{1}{2}$ unit long, $\frac{1}{2}$ unit wide, and $\frac{1}{2}$ unit high.

1. Complete the chart below, to find the possible dimensions and volume of the large box.
(use the blocks to help you determine the dimensions of the large box)

When calculating volume, leave in fraction form.

Length **in units**	Width **in units**	Height **in units**	Volume **in cubic units**

2. What is the volume of the small box in cubic units? _______ (leave the volume as a fraction)

3. How can you find the volume of the large box using the volume of the small boxes?

____

We Pack and Ship

Part B:

Another customer brings you 24 of the same size boxes to be packaged in one large box. The boxes must be packed with no gaps or extra room. Each box is $\frac{1}{2}$ unit long, $\frac{1}{2}$ unit wide, and $\frac{1}{2}$ unit high.

1. Complete the chart below, to find the possible dimensions and volume of the large box. (use the blocks to help you determine the dimensions of the large box)

When calculating volume, leave in fraction form.

Length **in units**	Width **in units**	Height **in units**	Volume **in cubic units**
$\frac{1}{2}\ unit$	$\frac{1}{2}\ unit$	$12\ units$	3 cubic units
$1\ unit$	$\frac{1}{2}\ unit$	$6\ units$	3 cubic units
$1\frac{1}{2}\ units$	$\frac{1}{2}\ unit$	$4\ units$	3 cubic units
$2\ units$	$1\ unit$	$1\frac{1}{2}\ units$	3 cubic units
$3\ units$	$1\ unit$	$1\ unit$	3 cubic units

2. What is the volume of the small box? $\frac{1}{8}$ cubic units

3. How can you find the volume of the large box using the volume of the small boxes?

   The volume of the small box is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ cubic units

   There are 24 small boxes

   $V = 24 \times \frac{1}{8} = 3\ cubic\ units$