IS NAEP MATH OUT OF STEP WITH THE STATES?

Introduction

Over the past few years nearly all states have made significant changes to their mathematics standards in an effort to better align with college- and career-ready expectations. At the same time, the National Assessment of Educational Progress (NAEP), seen as the "nation's report card," continues to play a significant role in providing a common metric by which states may compare themselves to each other. Expectations are changing, so it is important to consider the possibility of the misalignment of NAEP to the shifting goals in mathematics education in the U.S.

Several recent analyses have focused on the potential misalignment between the NAEP framework and the Common Core State Standards for Mathematics (CCSS-M). That is an understandable way to frame an analysis, especially since 42 states have adopted and are implementing the Common Core. However, in today's environment that framing carries a political colorization that is neither necessary nor helpful. NAEP is not intended to be responsive to any particular set of standards, and needs to be kept out of political debates about the Common Core.

There are two important perspectives to consider regarding the alignment of the NAEP framework[1] to currently used state standards. One perspective is to examine the topics that are assessed by NAEP earlier than they appear in state standards. The second is to consider topics that are required by the states but not assessed by NAEP. The first perspective outlines mismatches where states are held accountable for material they intentionally do not teach by the grade level, the second outlines mismatches where the NAEP is not sensitive to topics that receive instructional investment.

As the analysis below shows, the NAEP framework is out of step in key ways with what math educators in every state are expected to teach, especially at Grade 4, including in states that have never adopted the Common Core and in some that have recently replaced the Common Core. Further, data from NAEP's surveys of Grade 4 teachers show that the misalignment in math content between state standards and NAEP assessments corresponds to content that teachers report they are deemphasizing. In Grade 8 teachers report an increased emphasis on algebra, though the NAEP may be in danger of missing that important shift.

As a result, we believe it is time for the National Assessment Governing Board to review and revise where necessary the NAEP framework and the Grade 4 and Grade 8 assessments administered in each of the states, so that NAEP retains its important role as the gold standard for assessing and reporting on the progress of U.S. students nationwide.

Analysis

There is a growing body of work exploring the alignment of the NAEP to the CCSS-M.[2] With insight and results from these analyses, this report further explores NAEP alignment issues in CCSS-M states as well as states that never adopted the CCSS-M and those that have replaced the CCSS-M. Specifically, this report considers the mathematics standards in Minnesota, Virginia, Texas, Indiana, Nebraska, South Carolina, Oklahoma, and Tennessee. Minnesota, Virginia, and Texas never adopted the CCSS-M, while Indiana, South Carolina, Oklahoma, and Tennessee adopted, then replaced, the CCSS-M. Additionally, we present the nationwide shifts in heavy emphasis, reported by teachers and disaggregated by content area, as found in the NAEP data.

Grade 4

In Grade 4 there are clear issues where topics are assessed by NAEP, even though they have not yet been taught by that grade level. Table 1 provides a number of examples, along with when the topics are first introduced in different state standards. It is clear that issues of misalignment are by no means restricted to states that have adopted the Common Core State Standards for Mathematics (CCSS-M).

[1] National Assessment Governing Board, 2014

[2] Hughes, Daro, Holtzman and Middleton, 2013, Daro, Hughes and Stancavage, 2015, and Zimba, 2015

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SELECT NAEP GRADE 4 TOPICS AND THE GRADE LEVEL AT WHICH THOSE TOPICS ARE FIRST ADDRESSED IN THE STATES.

NAEP Grade 4 Objective[3]	Topic (required, but not necessarily a full match)	Grade where the topic is first addressed by each set of standards.[4]								
		MN 2007	VA 2009	CCSS 2010	TX 2012	IN 2014	NEB 2015	SC 2015	OK 2016	TN 2016
Add and subtract: Decimals through hundredths.	Add/subtract decimals	5	4	5	4	5	5	5	5	5
Use simple ratios to describe problem situations.	Ratio	6	6	6	6	6	6	6	6	6
Identify the images resulting from flips (reflections), slides (translations), or turns (rotations).	Transformational geometry	4	4	8	8	8	8	8	6	8
Graph or interpret points with whole number or letter coordinates on grids or in the first quadrant of the coordinate plane.	Graph points on the coordinate plane	5	6	5	5	5	5	5	5	5
For a given set of data, complete a graph (limits of time make it difficult to construct graphs completely). (Pictographs, bar graphs, circle graphs, line graphs, line plots, tables, and tallies.)	Circle Graphs	7	6	N/A	7	4[5]	7	N/A	7	6
Given a set of data or a graph, describe the distribution of data using median, range, or mode.[6]	Central Measures	5	5	6	6	5	6	6	5	6
Determine a simple probability from a context that includes a picture.	Probability	6	4	7	7	7	7	7	7	7
List all possible outcomes of a given situation or event.	Outcomes/ Sample Space	6	5	7	7	7	7	7	6	8
Recognize or describe a relationship in which quantities change proportionally.	Proportionality	7	7	7	6	6	7	7	7	7
Compare two sets of related data.	Data	7	9	7	7	7	6	7	9	7

[3] NAEP has nothing like the CCSS mathematical practices. These types of practices are found not only in the CCSS, but also VA, TX, and NE.

[4] This table is adapted from Zimba, 2015.

[5] The IN standard, 4.DA.3: *Interpret data displayed in a circle graph*, requires only interpretation and not completion of a circle graph.

[6] CCSS and MN do not mention mode.

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To illustrate, consider that the Grade 4 NAEP expects students to be able to recognize or describe a relationship in which quantities change proportionally. However, work with proportionality is first found in either Grade 6 or Grade 7 even in non-Common Core states, such as in the examples below:

- **Indiana, Grade 6:** Use variables to represent two quantities in a proportional relationship in a real-world problem; write an equation to express one quantity, the dependent variable, in terms of the other quantity, the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. (6.AF.10)

- **Minnesota, Grade 7:** Understand that a relationship between two variables, x and y, is proportional if it can be expressed in the form $y / x = k$ or $y = kx$. Distinguish proportional relationships from other relationships, including inversely proportional relationships (xy = k or y = k/x). (7.2.1.1)

- **South Carolina, Grade 7:** Identify and model proportional relationships given multiple representations, including tables, graphs, equations, diagrams, verbal descriptions, and real-world situations. (7.RP.2)

Two of these NAEP topics, addition and subtraction of decimals and ratio and proportionality, are even misaligned with the recommendations of the 2008 National Mathematics Advisory Panel:[7]

- "By the end of Grade 5, students should be proficient with comparing fractions and decimals and common percent, and with the addition and subtraction of fractions and decimals."

- "By the end of Grade 7, students should be able to solve problems involving percent, ratio, and rate and extend this work to proportionality."

The Study of the Alignment of the 2015 NAEP Mathematics Items at Grades 4 and 8 to the Common Core State Standards (CCSS) for Mathematics[8] reviewed all of the mathematics items used in the 2015 NAEP. The study concluded that roughly 21% of the items on the Grade 4 NAEP do not clearly match the CCSS-M expectations at Grade 4. Given that the vast majority of states have adopted the CCSS-M, the mismatch on Data Analysis is noteworthy as *less than half* (47%) of the items in the pool fall within the bounds of the CCSS with certainty. The mismatches in Algebra and Geometry are also noteworthy, where only 62% and 68% (respectively) of the items aligned with certainty to the CCSS. According to the report, the breakdown of items in the Grade 4 pool that matched the CCSS at or below Grade 4 is shown below:[9]

NUMBER AND PERCENT OF ITEMS ALIGNED AND MISALIGNED AT GRADE 4.

	In CCSS at or below NAEP Grade 4	Uncertain alignment or not in CCSS at or below NAEP Grade 4
Data analysis, statistics and probability	9 (47%)	10 (53%)
Algebra	13 (62%)	8 (38%)
Geometry	15 (68%)	7 (32%)
Number properties and operations	55 (90%)	6 (10%)
Measurement	26 (96%)	1 (4%)
Total	118 (79%)	32 (21%)

[7] National Mathematics Advisory Panel, 2008, p.20.

[8] Daro, Hughes and Stancavage, 2015

[9] Based on the table on p.8 of Daro, et al, 2015.

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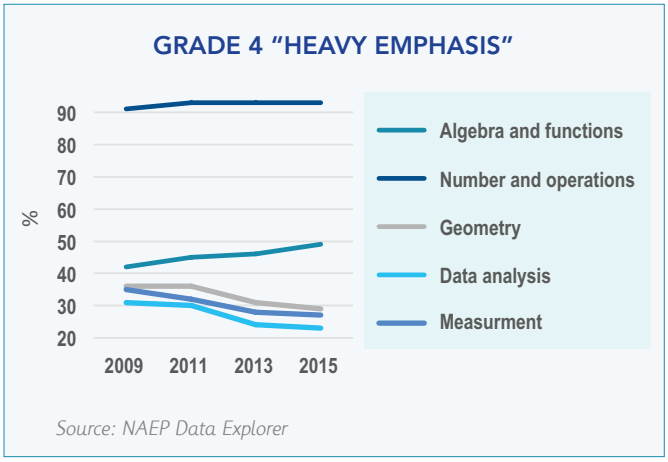
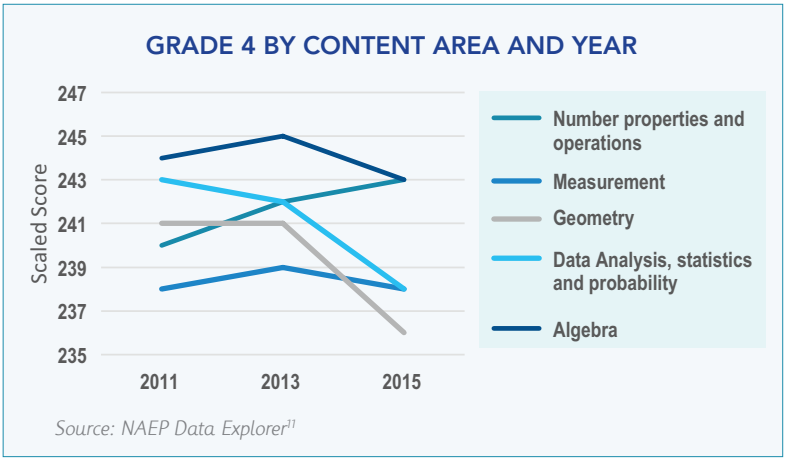
In addition, the NAEP also provides data related to how teachers claim to emphasize those content areas. The chart and table below indicate clear national trends in the "heavy emphasis" on the topics taught by Grade 4 teachers, from 2009 to 2015, and the percentages of students with those teachers. There is a strong upward trend in algebra in Grade 4, but there are downward trends in the emphasis placed on geometry, data analysis, and measurement. The NAEP tells us that teachers appear to be in the process of shifting what they emphasize.

Consider data analysis as a topic in Grade 4.[10] The NAEP framework includes data topics that are not being taught in any of the considered states, and teachers are indicating a reduction of emphasis on data. Over half of the data items for 2015 do not align with certainty to the expectations in the CCSS.

The mismatches described above largely coincide with the two NAEP content areas of (a) geometry and (b) data analysis, statistics, and probability. Recent scores on NAEP, when considered nationally by content area, show scores in those areas to be in steep decline. Further analysis might explore why the increased emphasis in algebra accompanies an overall drop in algebra scores:



Source: NAEP Data Explorer



Source: NAEP Data Explorer[11]

PERCENT OF STUDENTS WITH TEACHERS INDICATING "HEAVY EMPHASIS.

Grade 4 topic	2009	2011	2013	2015	Change from 2009-2015
Algebra and functions	42%	45%	46%	49%	+7
Number and operations	91%	93%	93%	93%	+2
Geometry	36%	36%	31%	29%	-7
Data analysis	31%	30%	24%	23%	-8
Measurement	35%	32%	28%	27%	-8

Source: NAEP Data Explorer

[10] Data analysis, statistics, and probability made up 13% of the 2015 NAEP, according to https://nces.ed.gov/nationsreportcard/mathematics/distributequest.aspx

[11] Average scale scores, grade 4 by all students, year and jurisdiction, National.

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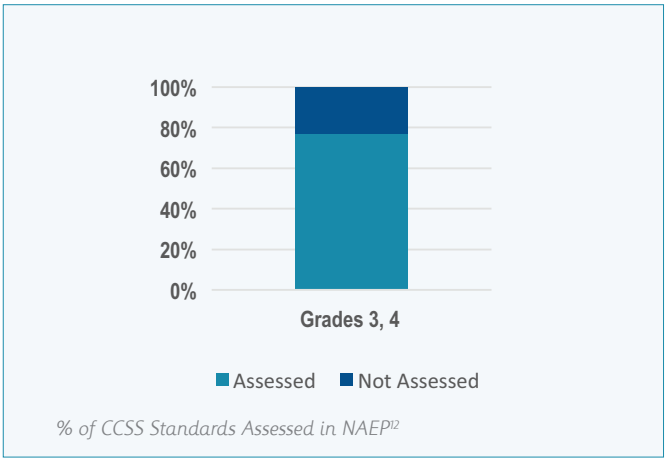
It is also relevant to consider content taught that is not assessed by NAEP, though this part of the story for Grade 4 is not as clear. From a CCSS perspective, the *Study of the Alignment of the 2015 NAEP Mathematics Items at Grades 4 and 8 to the Common Core State Standards (CCSS) for Mathematics* found that 23% of all CCSS for Grades 3 and 4 went untested in the NAEP items.



% of CCSS Standards Assessed in NAEP[12]

Grade 8

Instances of topics found in the NAEP framework, which are not in the CCSS, are not as pronounced in Grade 8 as in Grade 4. In fact the *Study of the Alignment Between the NAEP Mathematics Framework and the Common Core State Standards for Mathematics (CCSS-M)* claims that "the NAEP Grade 8 objectives in the content areas Number Properties and Operations, Algebra, and Geometry have very good coverage in the CCSS-M in Grade 6 through Grade 8. Gaps in coverage in the CCSS-M for NAEP Grade 8 objectives appear in the c ontent areas Measurement and Data Analysis, Statistics, and Probability."[13] The study points out, however, that in some cases the alignments are found several years prior in the CCSS and also indicates concerns with issues of mismatches in specificity and conceptual understanding.

As in Grade 4, there are many alignment issues when NAEP items are compared to the CCSS. The table below, based on the *Study of the Alignment of the 2015 NAEP Mathematics Items at Grades 4 and 8 to the Common Core State Standards (CCSS) for Mathematics*, indicates the alignment of items in the Grade 8 item pool that matched the CCSS at or below Grade 8 as shown.[14] As in the framework study, a key alignment issue is found in the topic of data analysis:

NUMBER AND PERCENT OF ITEMS ALIGNED AND MISALIGNED AT GRADE 8.

	In CCSS at or below NAEP Grade 8	Uncertain alignment or not in CCSS at or below NAEP Grade 8
Data analysis, statistics and probability	17 (74%)	6 (26%)
Geometry	20 (80%)	5 (20%)
Algebra	38 (84%)	7 (16%)
Number properties and operations	30 (97%)	1 (3%)
Measurement	26 (100%)	0 (0%)
Total	131 (87%)	19 (13%)

[12] Based on Daro, et al, 2015, p. 9.
[13] Hughes, et al, 2013, p. 50
[14] Based on the table on p.11 of Daro, et al, 2015.

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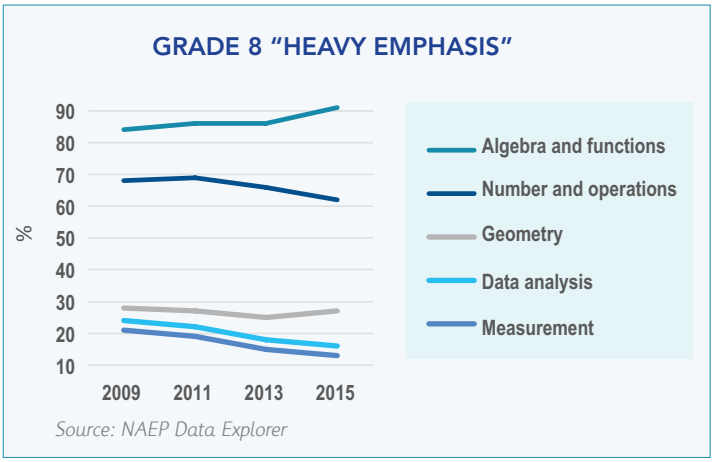
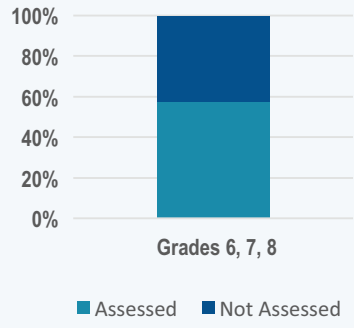
As with Grade 4, the NAEP report also provides data related to Grade 8 content areas and how the teachers claim to emphasize them. The chart and table below indicate specific trends in the "heavy emphasis" by Grade 8 teachers from 2009 to 2015, and the percentage of students with those teachers. There are downward trends in data analysis and measurement and a strong upward trend in algebra.

From the other perspective, there are topics in the Grade 8 CCSS that are not clearly found in the Grade 8 NAEP framework. The *Study of the Alignment of the 2015 NAEP Mathematics Items at Grades 4 and 8 to the Common Core State Standards (CCSS) for Mathematics* found that 42% of all Grades 6, 7, and 8 CCSS standards went untested in the NAEP items.



Source: NAEP Data Explorer



% of CCSS Standards Assessed in NAEP[15]

PERCENT OF STUDENTS WITH TEACHERS INDICATING "HEAVY EMPHASIS."

Grade 8 topic	2009	2011	2013	2015	Change from 2009-2011
Algebra and functions	84%	86%	86%	91%	+7
Number and operations	68%	69%	66%	62%	-6
Geometry	28%	27%	25%	27%	-1
Data analysis	24%	22%	18%	16%	-8
Measurement	21%	19%	15%	13%	-8

Source: NAEP Data Explorer

[15] Based on Daro, et al, 2015, p. 14.

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This list below shows some of the major CCSS topics[16] the NAEP framework appears to miss:

CCSS major content not found at grade level in the NAEP framework	
Grade 8	• Know and apply the properties of integer exponents to generate equivalent numerical expressions (see 8.EE.1) • Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$ (see 8.EE.2) • Perform operations with numbers expressed in scientific notation (see 8.EE.4) • Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b* (see 8.EE.6) • Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions (see 8.EE.7a) • Solve linear equations that require using the distributive property and collecting like terms (see 8.EE.7b) • Analyze and solve pairs of simultaneous linear equations (see 8.EE.8) • Compare properties of two functions each represented in a different way (see 8.F.2) • Describe qualitatively the functional relationship between two quantities by analyzing a graph. Sketch a graph that exhibits the qualitative features of a function that has been described verbally (see 8.F.5)

Some of this content, while not included in the NAEP framework, does appear in other states. The alignments are not universal, but more recent standards tend to emphasize some of these algebraic topics in Grade 8.

CCSS-M (Major, Grade 8) not in NAEP	MN 2007	VA 2009	TX 2012	IN 2014	NEB 2015	SC 2015	OK 2016	TN 2016
Apply integer exponents to generate equivalent numerical expressions (see 8.EE.1)	8	N/A	N/A	8	8	8	8	8
Perform operations with in scientific notation (see 8.EE.4)	8	N/A	N/A	8[17]	8	8	8	8
Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions (see 8.EE.7a)	N/A	N/A	N/A	8	8	8	8	8
Solve linear equations that require using the distributive property and collecting like terms (see 8.EE.7b)	8	8[18]	8[18]	8	7	8	7	8
Analyze and solve pairs of simultaneous linear equations (see 8.EE.8)	8	N/A	8[19]	8[20]	N/A	8	N/A	8

[16] Student Achievement Partners, 2013

[17] The IN standard (8.EE.4) is a bit unclear with the expectation to perform operations. The standard seems to go beyond the NAEP, though, by requiring solving problems involving scientific notation.

[18] Not necessarily fully aligned to CCSS but goes beyond NAEP.

[19] The TX expectation (8.b.9) only requires students to identify and verify the solution to two simultaneous linear equations from a graph.

[20] The IN standard (8.AF.8) only requires that students "[a]pproximate the solution of a system of equations by graphing and interpreting the reasonableness of the approximation." Algebraic solutions are not required.

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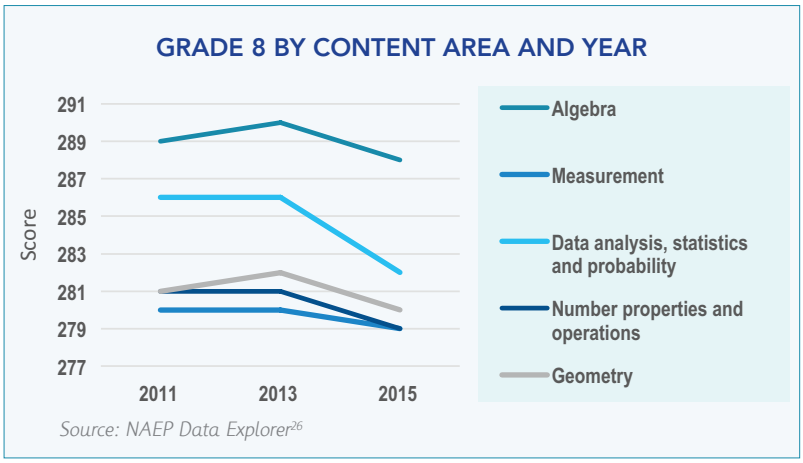
As shown in the table above, since 2014 state standards in non-Common Core states may also be making such a shift in emphasis of Grade 8 algebra topics. It is possible, however, that NAEP is not sensitive to increased expectations even though these topics receive instructional investment. The potential Grade 8 mismatch between NAEP and practice is also highlighted in the 2013 *Study of the Alignment Between the NAEP Mathematics Framework and the Common Core State Standards for Mathematics (CCSS-M)*. The study found that the "CCSS-M reflect the migration of Algebra I content to lower grades in the United States over the last two decades."[21] The framework study, corresponding to the information above, also indicated that for the Grade 8 NAEP, the problem is particularly acute in the algebraic handling of expressions and equations:

> "Expressions and Equations (algebra) is one content domain in the CCSS-M for which students may be learning mathematics that goes untested and undetected by NAEP at Grade 8. This is perhaps the most dangerous risk to the NAEP mission, given the national priority on algebra for all. It is fundamental to NAEP's mission that its assessments be able to detect progress in this high-priority domain. By not testing what the CCSS-M recommend should be taught, NAEP risks underestimating progress."[22]

It is possible, then, that the NAEP algebra scores are missing these national shifts, even though recent standards appear to address these shifts and teachers are reporting an increase in heavy emphasis on algebra in Grade 8. The framework study also determined that when compared to the NAEP, the CCSS provide:

1. More rigorous content in eighth-grade algebra and geometry
2. More extensive and systematic treatment of mathematical expertise (found in the Standards for Mathematical Practice)[23]
3. A more conceptual perspective on many mathematical topics, explicitly stating the mathematics to be understood rather than the type of problem to be solved
4. Some content taught at higher grades than is assessed in the fourth-grade NAEP assessment. For example, the study of proportional relationships is concentrated in Grades 6 and 7, and data sets and probability are taught in Grades 6 and 7, respectively.[24]

Finally, the chart below shows the trends of the NAEP Grade 8 scores by content area.[25] The sharpest drop (data analysis, statistics and probability) corresponds to the shifts in emphasis and item misalignment in the CCSS-M states. Further analysis might explore why the increased emphasis in algebra accompanies an overall drop in algebra scores:



Source: NAEP Data Explorer[26]

[21] Hughes, et al, 2013, p. 43

[22] Hughes, et al, p. 43

[23] There is no parallel to the Standards for Mathematical Practice in the NAEP framework. There are similar structures in the VA, TX, NE, and OK standards.

[24] Hughes, et al, p. 58

[25] NOTE: The NAEP Mathematics scale ranges from 0 to 500. Some apparent differences between estimates may not be statistically significant.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007, 2009, 2011, 2013, and 2015 Mathematics Assessments.

[26] Average scale scores, grade 8 by all students, year and jurisdiction, National.

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The conclusion of the *Study of the Alignment of the 2015 NAEP Mathematics Items at Grades 4 and 8 to the Common Core State Standards (CCSS) for Mathematics* said it well:

> "It has been 10 years since NAEP conducted a major review of its mathematics framework. Despite some uncertainties concerning the way that the CCSS will influence instruction over time, we believe that this is an appropriate moment for NAGB to review the framework in light of the CCSS as well as other states' college and career standards."[27]

[27] Daro, et al, p. 16

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