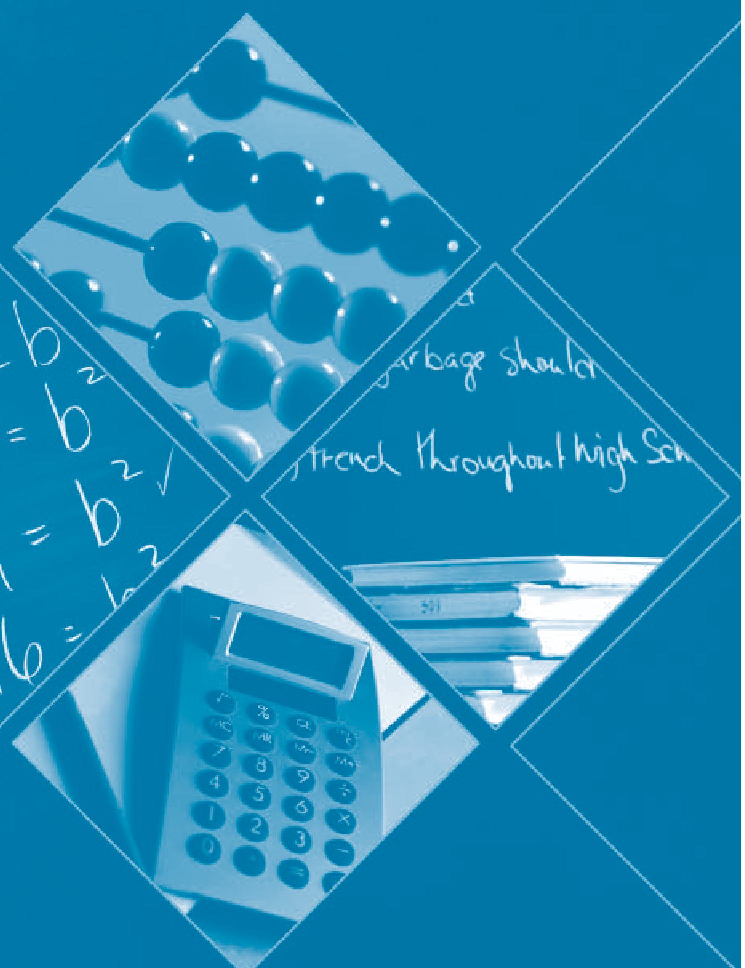
Student Name: ____________________

American Diploma Project

ALGEBRA I
End-of-Course Exam

PRACTICE TEST
COMMENTARIES



Created by the nation's governors and business leaders, Achieve, Inc., is a bipartisan non-profit organization that helps states raise academic standards, improve assessments and strengthen accountability to prepare all young people for postsecondary education and training, careers, and citizenship.

Pearson was awarded the contract to develop, deliver, score, and report the ADP Algebra I End-of-Course Exam. Through its Educational Measurement group, Pearson is the largest comprehensive provider of educational assessment products, services and solutions. As a pioneer in educational measurement, Pearson has been a trusted partner in district, state and national assessments for more than 50 years. Pearson helps educators and parents use testing and assessment to promote learning and academic achievement.

Background

The American Diploma Project (ADP) Network includes states dedicated to making sure every high school graduate is prepared for college and a career. In each state, governors, state superintendents of education, business executives and college and university leaders are working to restore value to the high school diploma by raising the rigor of high school standards, assessments and curriculum, and better aligning these expectations with the demands of postsecondary education and careers.

In May 2005, leaders from several of the ADP Network states began to explore the possibility of working together, with support from Achieve, to develop a common end-of-course exam at the Algebra II level. These states were planning to require or strongly encourage students to take an Algebra II level course in order to better prepare them for college and careers, as Algebra II or its equivalent serves as a gateway course for higher education and teaches quantitative reasoning skills important for the workplace. State leaders recognized that using a common end-of-course test would help ensure a consistent level of content and rigor in classes within and across their respective states. They also understood the value of working collaboratively on a common test: creating a high quality test faster and at lower cost to each state while also allowing comparison of student performance and progress with one another. To date, 15 states are part of this ADP Assessment Consortium initiative – Arkansas, Arizona, Florida, Hawaii, Indiana, Kentucky, Maryland, Massachusetts, Minnesota, New Jersey, North Carolina, Ohio, Pennsylvania, Rhode Island and Washington.

The development of the Algebra I end-of-course exam was a natural extension of this effort and was designed to support the goals of the Algebra II initiative. Leadership for content design and format was provided by a subset of the state content leaders involved in the development of the Algebra II end-of-course exam.

As an extension of the ADP Algebra II End-of-Course Exam, the ADP Algebra I End-of-Course Exam serves similar, parallel purposes:

1. ***To improve curriculum and instruction*** while *ensuring consistency within and across states***.** The exam will help classroom teachers focus on the most important concepts and skills in an Algebra I, or equivalent, class and identify areas

where the curriculum needs to be strengthened. For schools administering both exams, the Algebra I End-of-Course Exam will compliment the Algebra II End-of-Course Exam and will help ensure a compatible, consistent and well-aligned Algebra curriculum. Once standards are set, teachers will get test results back within three weeks of when the exam is administered, which will provide sufficient time to make the necessary adjustments for the next year's course.

2. ***To help high schools determine if students are ready for rigorous higher level mathematics courses.*** Because the test is aligned with the ADP mathematics benchmarks, it will measure skills students need to succeed in mathematics courses beyond Algebra I. High schools will be able to use the results of the exam to tell Algebra I students, parents, teachers and counselors whether a student is ready for higher level mathematics, or if they have content and skill gaps that need to be filled before they enroll in the next mathematics class in their high school's course sequence. This information should help high schools better prepare their students for upper level mathematics, which might include passing high school exit exams or state mathematics graduation exams. This will reduce the need for multiple retakes of courses or exams needed to graduate, hopefully avoiding remedial courses designed to review Algebra I skills and concepts.

3. ***To compare performance and progress among the participating states.*** Having agreed on the content expectations for courses at the Algebra I level, states are interested in tracking student performance over time. Achieve will issue a report each year comparing performance and progress among the participating states. This report will help state education leaders, educators and the public assess performance, identify areas for improvement and evaluate the impact of state strategies for improving secondary math achievement.

The Algebra I End-of-Course Exam will consist of Algebra I skills and concepts that are typically taught in an Algebra I course, which will be taken by students across participating states. States not part of the development group may also decide to purchase and administer this exam. The exam may be administered at any point in a student's course of studies from middle school through high school once a course in Algebra I or its equivalent is completed. The Exam Standards can be found at http://www.achieve.org/AlgebraITestOverview.

Algebra I level curriculum: Modeling and problem solving are at the heart of the curriculum at the Algebra I level. Mathematical modeling consists of recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution and interpreting the solution in the context of the original problem. Students must be able to solve practical problems, representing and analyzing the situation using symbols, graphs, tables or diagrams. They must effectively distinguish relevant from irrelevant information, identify missing information, acquire needed information and decide whether an exact or approximate answer is necessary, with attention paid to the appropriate level of precision. After solving a problem and interpreting the solution in terms of the context of the problem, the students must check the reasonableness of and devise independent ways of verifying the results.

It is also the case that curriculum at the Algebra I level will include content and processes not included on the Algebra I End-of-Course Exam, as some are not easily assessed by an exam of this nature. Problems that require extended time for solution should also be addressed in the Algebra I level classroom, even though they cannot be included in this end-of-course exam.

Algebra I level classroom practices: Effective communication using the language of mathematics is essential in a class engaged in Algebra I level content. Correct use of mathematical definitions, notation, terminology, syntax and logic should be required in all work at the Algebra I level. Students should be able to translate among and use multiple representations of functions fluidly and fluently. They should be able to report and justify their work and results effectively. To the degree possible, these elements of effective classroom practice are reflected in the Algebra I End-of-Course Exam content standards.

The Algebra I End-of-Course Exam: The Algebra I End-of-Course Exam covers a range of algebraic topics. Successful students will:

- demonstrate conceptual understanding of the properties and operations of real numbers with emphasis on ratio, rates and proportion, as well as numerical expressions containing exponents and radicals;

- operate with polynomial expressions, factor polynomial expressions and use algebraic radical expressions;
- analyze, represent and graph linear functions including those involving absolute value and recognize and use linear models;
- solve and graph linear equations and inequalities and then use them to represent contextual situations;
- solve systems of linear equations and model with single variable linear equations, one- or two-variable inequalities, or systems of equations;
- demonstrate facility with estimating and verifying solutions of linear equations, making use of technology where appropriate to do so;
- represent simple quadratic functions in multiple ways and use quadratic models, as well as solve quadratic equations;
- make connections to algebra through the interpretation of linear trends in data, the comparison of data using summary statistics, probability and counting principles, and the evaluation of data-based reports in the media.

There are a variety of types of items developed for the exam that will assess this content, including some that cut across the objectives in a standard, thus requiring students to make connections and, where appropriate, solve rich contextual problems. The Algebra I End-of-Course Exam will include three types of items: multiple-choice items (worth 1 point each), short-answer items (worth 2 points each) and extended-response items (worth 4 points each). Items on the exam, in particular extended-response items, may address more than one content objective and benchmark within a standard. Each standard within the exam is assigned a priority, indicating the approximate percentage of points allocated to that standard on the exam:

- Operations on Numbers and Expressions 25%
- Linear Relationships 35%
- Non-linear Relationships 20%
- Data, Statistics and Probability 20%

Approximately 30 percent of the student's score will be based on the short-answer and extended-response items. Although the test is untimed, it is designed to take approximately 120 minutes, comprised of two 60-minute sessions, one of which will allow calculator use. However, some students may require – and should be allowed – additional time to complete the test.

Algebra I End-of-Course Exam calculator policy: The appropriate and effective use of technology is an essential practice in the Algebra I classroom. At the same time, students should learn to work mathematically without the use of technology. Computing mentally or with paper and pencil is required on the Algebra I End-of-Course Exam and should be expected in classrooms where students are working at the Algebra I level. It is, therefore, important that the Algebra I End-of-Course Exam reflect both practices. For purposes of the Algebra I End-of-Course Exam, students are expected to have access to a calculator for one of the two testing sessions and the use of a graphing calculator is strongly recommended. Scientific or four-function calculators are permitted but not recommended because they do not have graphing capabilities. Students should not use a calculator that is new or different for them on the exam but rather should use the calculator to which they are accustomed and use every day in their classroom work. For more information about technology use on the Algebra I End-of-Course Exam, see the ADP Algebra End-of-Course Exams Calculator Policy at www.achieve.org/AssessmentCalcPolicy.

It will be necessary to clear the calculator memory, including any stored programs and applications, on all calculators both before and after the exam. Please be advised that the clearing of the calculator memory may permanently delete stored programs or applications. Students should be told prior to the day the exam is administered to store all data and software they wish to save on a computer or a calculator not being used for the exam. In some states, an IEP or 504 Plan may specify a student’s calculator use on this exam. Please check with your state’s Department of Education for specific policies or laws.

ADP Algebra I Practice Test Item Commentaries

This practice test is intended to offer insight into the format and expectations of the ADP Algebra I End-of-Course Exam. Of course, no set of items could possibly exemplify all of the available information surrounding an exam such as this, and items on the actual test will vary from items provided on the practice test.

These sample solutions and commentaries are provided as guidance to teachers and students participating in the ADP Algebra I End-of-Course Exam. Please note that the format of and style of text in the solutions and commentaries does not reflect exactly how items appear in the test books.

Some of the items on this practice test have been selected because they illustrate a particular type of notation with which students should become familiar; some of the items attempt to show the intended assessment limit of a particular standard; still others demonstrate the use of diagrams, graphs or contextual problem situations.

Sample solutions and commentary are provided as guidance and should not be considered exhaustive. On the actual Algebra I End-of-Course Exam, the student's choice of solution method is limited by the placement of the items in the calculator or non-calculator session of the exam. The solutions presented in this document are aligned with the session in which an item appears. However, students should be able to solve test problems in multiple ways, with and without a calculator.

For each item, this commentary will include the following:

- Item Type
- Calculator (permitted or NOT permitted)
- Benchmark
- Correct Answer (multiple choice)
- Explanation of Distractors (multiple choice) to indicate common errors that students could make
- Instructional Implications
- Sample Solution (methods)

Item 1

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: L1.a – Recognize, describe and represent linear relationships using words, tables, numerical patterns, graphs and equations. Translate among these representations.

Correct Answer: D

Explanation of Distractors:

Rationale A	Confuses exponent with multiplication, or thinks this is linear. Function is exponential since the variable is in the exponent.
Rationale B	Confuses exponent with multiplication or addition. Fails to recognize that the degree of the variable is not one.
Rationale C	Confuses quadratic with linear. Fails to recognize that the degree of the variable is not one.

Instructional Implications: A linear function is one that is either a constant function or a polynomial function with degree of one. Linear relationships can be expressed generally as:

$f(x) = c$, where *c* is a constant, or
$f(x) = ax + b$, where *a* and *b* are both constants.

Sample Solution: This question requires students to pay close attention to and interpret the location of the variable and the numerals in the expression of the functions. Expressions with a variable in the exponent are exponential. The other three expressions are all polynomials, but only the function $f(x) = x + 3$ is linear because the exponent of x is one.

Item 2

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: D1.c – Evaluate the reliability of reports based on data published in the media.

Correct Answer: A

Explanation of Distractors:

Rationale B	Chooses a correct graph display for the data but one that is not appropriate for the article. The break on the *y*-axis makes the cost appear to start out low, but the scale chosen for the y-axis allows the cost to appear to rise significantly over time.
Rationale C	Chooses a correct graph display for the data but one that is not appropriate for the article. The cost appears high because the graph is higher above the x-axis than the graph in distractor A, B, or D. Also the scale chosen for the y-axis allows the cost to appear to rise somewhat significantly over time.
Rationale D	Chooses a correct graph display for the data but one that is not appropriate for the article. The scale chosen for the y-axis allows the cost to appear to rise more slowly over time than the cost rises in distractors B and C but more quickly than the cost rises in answer A.

Instructional Implications: While choices in graphical representations are somewhat subjective, it is important that students develop the ability to analyze the visual impact and message of such a representation. Students should be encouraged to look carefully at the title of the article and evaluate the representations in light of the message it conveys.

Sample Solution: In this example, the vertical scale (*y*-axis) affects the impression of how fast the tuition is increasing. The distance the graph appears to be above the *x*-axis affects the impression of how low the tuition is. One goal for selecting a graph for the headline is to select a graph with a *y*-axis scale that allows the cost of all 10 years of tuition to look very similar in value. This scale should make the spread from $6,000 to $10,000 be a relatively small part of the total dollar values displayed, giving the impression that tuition costs have been fairly consistent. A second goal is to have the data appear in the lower portion of the graph, giving the impression that costs are fairly low. Choice A, with a *y*-scale that is continuous from 0 to 30, best accomplishes both of these goals.

Item 3

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: L1.c – Graph the absolute value of a linear function and determine and analyze its key characteristics.

Correct Answer: C

Explanation of Distractors:

Rationale A	The slope is correct, but the vertex of this function lies on the x-axis at $(3, 0)$ instead of on the y-axis at $(0, -3)$ as shown on the graph.
Rationale B	This is a linear function since the absolute value does not include the variable.
Rationale D	The slope is correct, but the vertex of this function lies on the x-axis at $(2, 0)$ instead of on the y-axis at $(0, -3)$ as shown on the graph.

Instructional Implications: This problem focuses on the vertex of the absolute value function. In the graph, the lowest or highest point of the "V" is the vertex. A key concept is how the vertex can be identified from both the graphical and symbolic representation of the absolute value function. It is helpful to have students analyze the transformations that have been applied to the parent function, $f(x) = |x|$. Experience with many such graphs will begin to lay the groundwork for the generalization of the effect of transformations on functions introduced in the future.

Sample Solutions:

Analytic Approach: The slope of the right branch of the graph is $\frac{3}{2}$; therefore, determining the slope of the choices given is one approach to eliminating some of the distractors. Unfortunately, in this case all choices describe functions where at least one branch, or the entire function, has a slope $\frac{3}{2}$, and so slope is not a good discriminator for this problem.

In the general form of the absolute value function, $f(x)=|a(x-h)|+k$, an a that is not 1, changes the slopes of the branches; an h that is not 0 translates the vertex horizontally; and a k that is not 0 translates the vertex vertically. On the graph, h = 0, so distractors A and D can be eliminated because h = 3 for both of these functions. In addition, substituting 0 for x in either of these functions yields an output of 3, and so the point $(0,-3)$ does not lie on the graph of either of these functions. Distractor B is a linear function; the function $f(x)=\left|\frac{3}{2}\right|x-3$ is the same as the function $f(x)=\frac{3}{2}x-3$. While the function passes through the point $(0,-3)$, that point is not the vertex.

The equation $f(x)=|a(x-h)|+k$ represents an absolute value function with slopes $\pm a$ and vertex (h, k). The graph appears to have its vertex at $(0,-3)$ and to pass through the two points $(2, 0)$ and $(-2, 0)$. The slope of the right branch of the graph is $\frac{-3-0}{0-2}=\frac{3}{2}$. The slope of the left branch of the graph is $\frac{-3-0}{0-(-2)}=-\frac{3}{2}$. Substituting, the function in this problem is Distractor C, $f(x)=\left|\frac{3}{2}(x-0)\right|+(-3)=\left|\frac{3}{2}x\right|-3$.

<u>Transformational Approach:</u> The graph of the function $f(x)=|x|$ has a vertex at the origin and no x-intercepts. To obtain the graph given in this problem, $f(x)=|x|$ has been translated down three units and each of its points, (x, y) has been mapped to a point whose x-value is two-thirds of the related x-value of the parent function, $\left(\frac{2}{3}x, y\right)$ or, alternatively, mapped to a point whose y-value is $\frac{3}{2}$ the original y-value for any given x. In either case, this is represented symbolically by $f(x)=\left|\frac{2}{3}x\right|-3$.

Item 4

<u>Item Type</u>: Multiple Choice

Calculator: NOT Permitted

Benchmark: O1.d – Use the properties of radicals to rewrite numerical expressions containing square roots in different but equivalent forms or to solve problems.

Correct Answer: B

Explanation of Distractors:

Rationale A	Multiplies the denominator by $\sqrt{3}$ but fails to multiply the numerator by the same factor. $\frac{\sqrt{2}+\sqrt{6}}{\sqrt{3}}\left(\frac{1}{\sqrt{3}}\right)=\frac{\sqrt{2}+\sqrt{6}}{3}$
Rationale C	Reduces incorrectly. $\frac{\sqrt{2}+\sqrt{6}}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=\frac{\sqrt{6}+\sqrt{18}}{3}=\frac{\sqrt{6}+3\sqrt{2}}{3}\neq\sqrt{6}+\sqrt{2}$
Rationale D	Reduces incorrectly by reducing only the last term of the numerator. $\frac{\sqrt{2}+\sqrt{6}}{\sqrt{3}}\neq\sqrt{2}+\frac{\sqrt{6}}{\sqrt{3}}\neq\sqrt{2}+\sqrt{2}\neq 2\sqrt{2}$

Instructional Implications: Success with this problem relies on several fundamental understandings that students should bring from prior mathematical study. First, it is important to recognize that multiplication or division of *both* the numerator and the denominator of a fraction by the same factor – that is multiplying or dividing a fraction by 1 – changes the representation but not the value of the fraction. Secondly, it is important to know that the goal of rationalizing the denominator is to multiply by some number that will yield a *square number* under the square root in the denominator; in this problem, both numerator and denominator may be multiplied by $\sqrt{3}$ to create a rational denominator $\sqrt{3}\cdot\sqrt{3}=\sqrt{9}=3$. Finally, application of the accepted order of operations and the distributive property is critical to the correct solution. The fraction bar, or *vinculum*, serves as a grouping symbol in a fraction and indicates that both terms in the numerator are to be divided by $\sqrt{3}$. This problem provides an opportunity to reinforce prior work with fractions in a more complex numerical setting.

Sample Solution: Multiplying by 1 expressed as $\frac{\sqrt{3}}{\sqrt{3}}$ yields:

$$\frac{\sqrt{2}+\sqrt{6}}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=\frac{\sqrt{6}+\sqrt{18}}{3}$$
$$=\frac{\sqrt{6}+3\sqrt{2}}{3}$$

Item 5

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: N1.a – Recognize, describe, represent and analyze a quadratic function using words, tables, graphs or equations.

Correct Answer: D

Explanation of Distractors:

Rationale A	Incorrectly factors the negative out of the first factor to obtain $f(x)=-(x+3)(x-5)$ and fails to change signs for the intercepts.
Rationale B	Determines correct intercepts but fails to recognize the effect of the negative in front of the *x* in the first factor on the direction the graph opens.
Rationale C	Correctly factors $f(x)=(-x+3)(x-5)$ to obtain $f(x)=-(x-3)(x+5)$ but fails to change signs for the intercepts.

Instructional Implications: Critical to the solution of this problem is the correct application of the distributive property, factoring out the negative from the first factor. Once the factoring has been accomplished, the relationship between the symbolic representation and the directionality of the graph – the negative sign indicates the parabola opens down – is important knowledge. Also critical is the relationship between the intercepts, or *zeros*, of a function and the solutions, or *roots*, of the related equation. Finding intercepts and factoring are both sources of frequent errors by students as they continue in mathematics.

Sample Solution: Factor out the negative from the first factor to get $f(x) = (-x+3)(x-5) = -(x-3)(x-5)$. If this were multiplied out, the leading coefficient, i.e., the coefficient of the x^2 term would be negative, indicating that the graph is a parabola that opens down.

To determine the *x*-intercepts of the function, where $y=0$, solve the related equation:

$$-(x-3)(x-5)=0.$$

$$\begin{aligned} -(x-3)(x-5) &= 0 \\ -(x-3) = 0 \ \text{ or } \ (x-5) &= 0 \\ x = 3 \ \text{ or } \ x &= 5 \end{aligned}$$

Given as ordered pairs, the *x*-intercepts are $(3, 0)$ or $(5, 0)$.

Item 6

Item Type: Short Answer

Calculator: NOT Permitted

Benchmark: N1.b – Analyze a table, numerical pattern, graph, equation or context to determine whether a linear, quadratic or exponential relationship could be represented. Or, given the type of relationship, determine elements of the table, numerical pattern or graph.

Correct Answer: $r=12$. To earn full credit, student solutions must include a reasonable written explanation and/or algebraic work that justifies the answer.

Instructional Implications: This item relies on knowledge that an exponential function increases by repeated multiplication of a common factor. To determine the value of *r* students need to identify that common factor and multiply the previous *y*-value by that common factor. Students should be reminded to check the progression of the *x*-values in the table to be sure that they are increasing by a common additive value. In this problem, that is the case, but focusing only on *y*-values where the *x*-values do not follow such a pattern will likely result in an incorrect answer.

Some students may be able to determine a specific exponential function that will yield the given input/output pairs indicated in the table. Students opting for this algebraic approach are focusing on defining the relationship between each *x*-value and its related *y*-value rather than on the relationship between *y*-values. This approach does not rely on a constant increase in *x*-values within the table.

<u>Sample Solutions</u>:

<u>Sample common factor approach</u>: The desired outcome of this problem is for the value of *r* to make the table a representation of an exponential function. Before determining a common factor between *y*-values, the progression of the *x*-values in the table needs to be checked to be sure that they are increasing by a common additive value. The *x*-values in this table increase by the common additive value of 1. Then, the common factor of the exponential function must be determined using the using the *y*-values in the table.

$$\frac{6}{3} = \frac{r}{6}$$
$$36 = 3r$$
$$r = 12$$

Sample explanation: As the *x*-value increases by one, the *y*-value is multiplied by 2. $6 \cdot 2 = 12$ so $r = 12$.

<u>Sample algebraic approach</u>: The problem calls for an exponential function. A simple general form of an exponential function is given by $f(x) = a \cdot b^x$.

$$\begin{array}{lll} f(0) = a \cdot b^0 = 3 & & f(1) = a \cdot b^1 = 6 \\ a \cdot 1 = 3 & \text{and} & 3 \cdot b = 6 \\ a = 3 & & b = 2 \end{array}$$

, so the function is $f(x) = 3 \cdot 2^x$.

Using this function, $f(2) = 3 \cdot 2^2 = 3 \cdot 4 = 12$, so $r = 12$.

Sample explanation: $r = 12$, the function represented in the table can be represented by the function $f(x) = 3 \cdot 2^x$, which would yield 12 at $f(2)$.

Item 7

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: L2.d – Solve systems of linear equations in two variables using algebraic and graphic procedures.

Correct Answer: A

Explanation of Distractors:

Rationale B	Confuses same slope as coincident lines.
Rationale C	Point satisfies the first equation but only works for the second if the negative is ignored.
Rationale D	Point satisfies the first equation but not the second.

Instructional Implications: When this system is solved algebraically, no point satisfies both equations. That means that this system of equations has no solution, indicating that the two lines they represent do not intersect. The equations describe parallel lines. It is important that when students solve this system algebraically, they need to be sure to check any potential solution in *both* equations. Students may also determine the correct response without solving the system of equations if they know how to compare slopes and *y*-intercepts directly from the symbolic form of the lines and recognize that lines having the same slope, regardless of the *y*-intercept, are parallel.

Sample Solutions:

Solution Set Approach: Solving the system of equations.

<u>Addition Method</u>	<u>Substitution Method</u>
$x + y = 6$ $y = -x + 2$ Rewrite the first equation and add to obtain: $y = -x + 6$ $\underline{y = -x + 2}$ $2y = 8$ $y = 4$ Substitute 4 for y in the first equation to obtain: $x + 4 = 6$ $x = 2$ Check this solution in the second equation to see that is does work. $4 = -2 + 2$ $4 = 0$ This system of equations has no solution indicating that the lines are parallel.	$x + y = 6$ $y = -x + 2$ Substitute $-x + 2$ in for y in the first equation to obtain: $x + (-x + 2) = 6$ $2 = 6$ Since this statement is false, there is no solution for the system of equations indicating that the lines are parallel.

<u>Graphical Approach</u>: Create a table of values for each equation.

$x + y = 6$

or

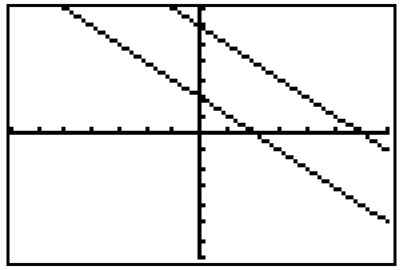
y = -x + 6

x	**y**
-1	7
0	6
2	4

$y = -x + 2$

x	**y**
-1	3
0	2
2	0

Then, sketch a graph of each equation on the same axes.



While it is not possible to tell for sure, it appears from the graph that the two lines are parallel. Checking that the slopes are the same and the y-intercepts are different will confirm that conclusion.

<u>Slope-intercept Approach</u>: To easily compare slopes, the equations can be rewritten:

$$y = -x + 6$$
$$y = -x + 2$$

Both of these lines have the same slope, -1, meaning that they either represent the same line or parallel lines. Because the *y*-intercepts differ, the lines are parallel.

Item 8

<u>Item Type</u>: Multiple Choice

<u>Calculator</u>: NOT Permitted

<u>Benchmark</u>: N2.b – Solve single-variable quadratic equations.

<u>Correct Answer</u>: D

Explanation of Distractors:

Rationale A	$4x^2 - 10x + 6 = 0$ erroneously factored to obtain $(4x+2)(x-3) = 0$ $4x + 2 = 0, \quad x - 3 = 0$ $4x = -2, \quad x = 3$ $x = -\frac{1}{2}, \quad x = 3$ This factoring would yield the correct coefficient for the x^2 and the x term but an incorrect sign for the constant term.
Rationale B	$4x^2 - 10x + 6 = 0$ erroneously factored to obtain $(2x+2)(2x-3) = 0$ $2x + 2 = 0, \quad 2x - 3 = 0$ $2x = -2, \quad 2x = 3$ $x = -1, \quad x = \frac{3}{2}$ This factoring would yield the correct x^2 term. The two linear terms, $-6x$ and $4x$, do not yield the correct middle term when added and the constant term is -6 instead of 6.
Rationale C	$4x^2 - 10x + 6 = 0$ erroneously factored to obtain $(4x-3)(x-2) = 0$ $4x - 3 = 0, \quad x - 2 = 0$ $4x = 3, \quad x = 2$ $x = \frac{3}{4}, \quad x = 2$ This factoring would yield the correct x^2 and constant term. The two linear terms, $-3x$ and $-8x$, do not yield the correct middle term when added.

Instructional Implications: Attention to the signs of the linear and constant terms of the quadratic is critical in correct factoring of a quadratic. It may be easier for students to first factor out the common constant factor, 2, or to divide the equation by 2 in order to minimize the number of options for factoring over the integers.

Sample Solution: Correctly factoring:

$$\begin{aligned}4x^2 - 10x + 6 &= 0\\ (2x-3)(2x-2) &= 0\\ 2x-3=0, \quad 2x-2&=0\\ 2x=3, \quad 2x&=2\\ x=\frac{3}{2}, \quad x&=1\end{aligned}$$

or

$$\begin{aligned}&4x^2 - 10x + 6 = 0\\ &2\left(2x^2 - 5x + 3\right) = 0\\ &\text{dividing by 2}\\ &(2x-3)(x-1) = 0\\ &2x-3=0, \quad x-1=0\\ &2x=3, \quad x=1\\ &x=\frac{3}{2}, \quad x=1\end{aligned}$$

Item 9

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: L2.c – Graph and analyze the graph of the solution set of a two-variable linear inequality.

Correct Answer: B

Explanation of Distractors:

Rationale A	Graph has the correct boundary line but shades the incorrect side of the line. This is the graph of $y < 2x - 10$.
Rationale C	Graph of the boundary line has a slope of -2 instead of 2. Shading is correct for the graph of $y > -2x - 10$.
Rationale D	Graph of the boundary line has a slope of -2 instead of 2. Shading is correct for the graph of $y < -2x - 10$.

Instructional Implications: Graphing an inequality in two variables requires that the boundary line be identified and graphed correctly. It is important for students to recognize and understand when to use each type of boundary line. In this case, the inequality is a strict inequality; therefore, the boundary line should be dotted. Graphing the line may be accomplished by creating a table of values or by rewriting the equation in slope-intercept form. The next step in graphing the inequality is to shade the correct side of the boundary line. To determine which side of the line should be shaded, a point that clearly does not lie on the line can be checked by substitution in the inequality. If the result yields a true statement, then the shading should be on the side that includes that point; if the result yields a false statement, then the shading should be on the opposite side of the line. Alternatively, the inequality can be rewritten as $2x - 10 < y$ or $y > 2x - 10$. This shows that the y-values to be shaded are always greater than the y-values of the line so the shading should be *above* the line.

<u>Sample Solution</u>: The boundary line for the inequality $2x - y < 10$ is $2x - y = 10$. Expressed in slope-intercept form the line is $y = 2x - 10$ making it easier to determine the slope of 2 and y-intercept $(0, -10)$. Only choices A and B fit these conditions. The point $(0, 0)$ lies in the shaded region of graph B but not in graph A. Substituting the coordinates of this point in the inequality yields:

$$\begin{aligned} 2x - y &< 10 \\ 2 \cdot 0 - 0 &< 10 . \\ 0 &< 10 \end{aligned}$$

This is a true statement indicating that the point *should be* part of the graph of the inequality as is the case in choice B.

Item 10

<u>Item Type</u>: Multiple Choice

<u>Calculator</u>: NOT Permitted

<u>Benchmark</u>: D.2.b – Apply probability concepts to determine the likelihood an event will occur in practical situations.

<u>Correct Answer</u>: A

<u>Explanation of Distractors</u>:

Rationale B	Determines the probability for rolling a 2 on the number cube correctly and multiplies by the probability that assumes that any number on the spinner is acceptable. $\left(\frac{4}{4}\right)\left(\frac{1}{6}\right) = \frac{4}{24} = \frac{1}{6}$

Rationale C	Determines an incorrect probability for rolling a 2 and multiplies by the probability that assumes that any number on the spinner is acceptable. $\left(\frac{4}{4}\right)\left(\frac{4}{6}\right)=\frac{16}{24}=\frac{2}{3}$
Rationale D	Determines the probability of spinning a 2 correctly and adds the probability of rolling any number on the spinner on the cube. $\frac{1}{4}+\frac{4}{6}=\frac{6}{24}+\frac{16}{24}=\frac{22}{24}$

Instructional Implications: It is necessary to assume, as stated on the ADP Algebra I End-of-Course Exam Expectations of Knowledge document, that both the spinner and the number cube are *fair.* That is, it is assumed that there is an equally likely chance of landing on any number on the number cube and that the spinner has an equally likely chance of landing in any direction. It is important for students to visualize a number cube and the number of sides it contains, especially since one is not pictured. Because these events are independent, the probabilities should be multiplied to determine the probability of them both happening.

Sample Solution: Recognize that the spinner has a one in four chance of landing on the 2 – 90° out of 360°, or $\frac{1}{4}$. Recognize that the die has a one in six, or $\frac{1}{6}$, chance of landing on the 2. The events are independent so the probability that both events occur is given by $\frac{1}{4}\cdot\frac{1}{6}=\frac{1}{24}$.

Item 11

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: N1.a – Recognize, describe, represent and analyze a quadratic function using words, tables, graphs or equations.

Correct Answer: C

Explanation of Distractors:

Rationale A	Student incorrectly assumes that all parabolas have a vertex at the origin.
Rationale B	Solves $0 = 2x^2 - 4x$ to determine the zeros and uses these zeros as the vertex. $2x^2 - 4x = 0$ $x(2x - 4) = 0$ $x = 0,\ \ x = 2$
Rationale D	Correctly determines the *x*-coordinate, but makes a sign error in the computation of the functional value. $x = \frac{-b}{2a} = \frac{-(-4)}{2(2)} = \frac{4}{4} = 1$ $f(x) = 2(1)^2 + 4(1) = 6$

Instructional Implications: Students will need to have some method for determining the *x*-coordinate of the vertex of the quadratic function either by finding the zeros of the function, then averaging them to find the *x*-coordinate or identifying the *x*-coordinate from the coordinates of the quadratic expression. The *x*-value found will be substituted into the function to find the *y*-coordinate of the vertex.

Sample Solutions:

Zeros Approach: To find the zeros, the function $f(x) = 2x^2 - 4x$ must be set equal to zero and then solved.

$$f(x) = 2x^2 - 4x$$
$$0 = 2x^2 - 4x$$
$$0 = 2x(x - 2)$$
$$x = 0, \quad x = 2$$

Then, the zeros are averaged, $\frac{0 + 2}{2} = 1$. This is the *x*-value of the vertex. This value is substituted into the function to find the *y*-value of the vertex, $f(1) = 2(1)^2 - 4(1) = 2 - 4 = -2$. The vertex is $(1, -2)$.

Coefficient Approach: For the function $f(x) = ax^2 + bx + c$, the x-coordinate of the vertex is given by the ratio $-\dfrac{b}{2a}$. So the vertex $(1, -2)$ is found by the calculation:

$$x = \frac{-b}{2a} = \frac{-(-4)}{2(2)} = \frac{4}{4} = 1$$
$$f(x) = 2(1)^2 - 4(1) = 2 - 4 = -2$$
.

Item 12

Item Type: Extended Response

Calculator: NOT Permitted

Benchmark: Extended-response items can be written to address multiple aspects of the standard. This particular item was written to the O standard, *Operations on Numbers and Expressions*, and mainly addresses the following benchmarks within the standard.

O.2.b – Add, subtract and multiply polynomial expressions with or without a context.

Correct Answer:

Part A: $(12x^2 + 41x + 35)$ sq. ft. To earn full credit, student solutions must include a reasonable written explanation and/or algebraic work that justifies the answer.

Part B: $(12x + 21)$ ft. and $(9x + 15)$ ft.

Part C: $(96x^2 + 328x + 280)$ sq. ft. To earn full credit, student solutions must include a reasonable written explanation and/or algebraic work that justifies the answer.

Instructional Implications: This item calls for students to apply their understanding that area is the result of multiplication to a situation involving algebraic expressions. The area of a rectangle as length-times-width should be very familiar to students. Once the binomial expressions are correctly identified as the factors of length and width, for Part A students must correctly multiply the binomials.

Students must remember that the linear unit of *feet* becomes *feet squared* when the binomials are multiplied. For Part B, students need to know that a scale factor of 3 means that all lengths in the figure are multiplied by 3, so both length and width are tripled. Part C does require some interpretation on the part of the student to understand that the *garden* is only the section *surrounding* the inner rectangle – the memorial site is *not* to be included. This requires that the expression for the area of the memorial site be subtracted from the expression for the area of the larger rectangle to determine the area of the garden.

Sample Solutions:

Part A: The length of the memorial site is given by $(4x+7)$ ft. The width of the memorial site is given by $(3x+5)$ ft. The area is given by:

$$\begin{aligned} A &= (4x+7)(3x+5) \\ &= \left(12x^2+41x+35\right)\text{sq. ft.} \end{aligned}$$

Part B: Since the scale factor is given as 3, each of the expressions for length and width need to be multiplied by 3 yielding $3(4x+7)=12x+21$ and $3(3x+5)=9x+15$, which represent respectively the length and width of the larger rectangle.

Part C: The garden area is the area of the large rectangle minus the area of the memorial site. The area of the larger rectangle is given by:

$$\begin{aligned} A &= lw \\ A &= (12x+21)(9x+15) \\ &= \left(108x^2+369x+315\right)\text{sq. ft.} \end{aligned}$$

Using this result and the area computed in Part A, the area of the garden is given by:

$$A_{total} - A_{memorial} = A_{garden}$$

$$\left(108x^2+369x+315\right)-\left(12x^2+41x+35\right)=\left(96x^2+328x+280\right)\text{sq. ft.}$$

Item 13

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: O1.c – Apply the laws of exponents to numerical expressions with integral exponents to rewrite them in different but equivalent forms or to solve problems.

Correct Answer: B

Explanation of Distractors:

Rationale A	Sets up correct division but expresses the number of decimal places in 2×10^{-10} incorrectly. $\dfrac{0.0001}{0.000000002}$
Rationale C	Multiplies the quantities instead of dividing but still subtracts the exponents. $\left(1 \times 10^{-4}\right)\left(2 \times 10^{-10}\right) \neq 2 \times 10^{-4+10}$ $\neq 2 \times 10^{6}$
Rationale D	Sets up the correct division but fails to account for the decimal point in 0.5 when computing the final exponent. $\dfrac{1 \times 10^{-4}}{2 \times 10^{-10}} = 0.5 \times 10^{-4+10}$ $\neq 5 \times 10^{6}$

Instructional Implications: To be successful with this item, students need to be familiar with how to read and express very large and very small numbers using scientific notation, and they need to be careful in the application of place value with respect to powers of 10. It is also necessary to recognize that determining the number of atoms that would fit across the diameter of a pin calls for division of the diameter of the pin by the diameter of the atom.

Sample Solution: The approximate number of atoms that will fit across the diameter of the pin is given by the quotient:

$$
\begin{aligned}
\frac{\text{Diameter of pin}}{\text{Diameter of atom}} &= \frac{1\times 10^{-4}}{2\times 10^{-10}} \\
&= 0.5\times 10^{-4+10} \\
&= 5\times 10^{-1}\times 10^{6} \\
&= 5\times 10^{5} \\
&= 500{,}000
\end{aligned}
$$

Item 14

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: L1.b – Describe, analyze and use key characteristics of linear functions and their graphs.

Correct Answer: A

Explanation of Distractors:

Rationale B	Lines are parallel.
Rationale C	Lines are coincident.
Rationale D	Lines are parallel.

Instructional Implications: In this item, students need to know that lines that intersect have non-equivalent slopes. Students should recognize that perpendicular lines are a unique form of intersecting lines and that lines can intersect without being perpendicular.

Sample Solution: The slope of the given line is $-\frac{1}{2}$. Distractors B, C and D all have the slopes that are equivalent to the given line. The slope of the line given in choice A is $-\frac{2}{3}$, and so that line will intersect the given line in only one point. That point *need not* be identified for this problem, but its coordinates may be determined to be $(12, -11)$ using either substitution or linear combination to solve the system of equations.

Item 15

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: D1.a – Interpret and compare linear models for data that exhibit a linear trend including contextual problems.

Correct Answer: C

Explanation of Distractors:

Rationale A	Disregards the fact that a customer may purchase more than just one ticket at the event.
Rationale B	Disregards the varying amounts spent by different customers.
Rationale D	Chooses a difference instead of an average, meaning that only the numerator of the slope is being considered.

Instructional Implications: Students need to understand the reference to "the line of best fit" and be able to express the meaning of the slope of that line in terms of the context of the problem. In addition, students need practice reading and interpreting what questions like this one are asking them to do. In this item, the fact that each successive data point represents the money collected after an *additional five* customers may be a stumbling block for students and should be discussed.

Sample Solution: In items such as this, students will need to read through the answer choices and choose the appropriate one. Distractor A neglects to take into account that the first sentence indicates that tickets and refreshments were sold for the fundraiser and included in the amount of money collected. When considering distractor B, students should recognize that a "line of best fit" is not an exact fit to every data point but a line that approximates all data points together. Answer choice C uses the interpretation of the slope of the line of best fit, that is, average amount of money spent per customer. However, distractor D uses only part of the definition of slope, that is, the difference in the y-values, the difference in the amount of money spent by any group of two customers.

Item 16

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: L2.e – Recognize, express and solve problems that can be modeled using single-variable linear equations; one- or two-variable inequalities; or two-variable systems of linear equations. Interpret their solutions in terms of the context of the problem.

Correct Answer: D

Explanation of Distractors:

Rationale A	Did not include Chris's contribution and subtracted 10 from right side of equation
Rationale B	Subtracted 10 from right side of equation.
Rationale C	Did not include Chris's contribution.

Instructional Implications: Two approaches are possible for this problem – one using a different variable for the number of skateboards painted by each of the two co-workers and one that uses a single variable to express these quantities. Students may benefit from seeing that either approach will yield the equation in choice D after only a few steps.

Sample Solutions:

Bivariate Approach	Single Variable Approach
Let x = the number of skateboards painted by Chris. Let y = the number of skateboards painted by Kim. The equation representing the total number of skateboards painted is $x+y=100$. The equations showing the relationship between the number painted by Chris and the number painted by Kim is $y=2x+10$.	Let x = the number of skateboards painted by Chris. Then $2x + 10$ = the number of skateboards painted by Kim. The total number of skateboards painted by both workers is 100 so solving the equation $$x+(2x+10)=100$$ $$3x+10=100$$ would yield the number of

Substituting for y in the first equation yields $$\begin{aligned} x + (2x + 10) &= 100 \\ 3x + 10 &= 100 \end{aligned}.$$ The equation that would be used to determine x, the number of skateboards Chris painted.	skateboards Chris painted, x.

Item 17

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: L2.a – Solve single-variable linear equations and inequalities with rational coefficients.

Correct Answer: B

Explanation of Distractors:

Rationale A	Distributes coefficient -3 incorrectly. $6 - 3(4x - 5) = 7$ is incorrectly equated to $$\begin{aligned} 6 - 12x - 15 &\neq 7 \\ -12x &\neq 16 \\ x &\neq -\frac{4}{3} \end{aligned}$$
Rationale C	Used the incorrect order of operations. $6 - 3(4x - 5) = 7$ is incorrectly equated to $$\begin{aligned} 3(4x - 5) &\neq 7 \\ 12x - 15 &\neq 7 \\ x &\neq \frac{11}{6} \end{aligned}$$

Rationale D	Used the incorrect order of operations. $6-3(4x-5)=7$ is incorrectly equated to $3(4x-5)\neq 13$ $12x-15\neq 13$ $12x\neq 28$ $x\neq \frac{7}{3}$

Instructional Implications: Correct application of the distributive property in situations similar to the one exemplified in this item benefit from implicit discussion and student practice. In addition, students will benefit from practice solving equations involving signed numbers.

Sample Solution:

$$\begin{aligned}6-3(4x-5)&=7\\6-12x+15&=7\\-12x&=-14\\x&=\frac{7}{6}\end{aligned}$$

Item 18

Item Type: Short Answer

Calculator: NOT Permitted

Benchmark: L1.d – Recognize, express and solve problems that can be modeled using linear functions. Interpret their solutions in terms of the context of the problem.

Correct Answer:

Part A: The coefficient 500 represents the mass (in grams) of one metal block.

Part B: The y-intercept represents the mass (in grams) of the cart when it is empty.

Instructional Implications: Students who have had many experiences interpreting a contextual situation and writing an expression or equation that will define a related problem in mathematical terms are likely to have more success working backwards to interpret aspects of the equation in light of the context. Asking students to complete a table of values and analyze the process may provide scaffolding for students who have difficulty with this process. Students may need to be reminded about the need to include units when answers are contextual in nature.

Sample Solution:

Part A: The functional value $f(b) = 500b + 5{,}500$ represents the mass of a cart loaded with *b* metal blocks. Each time one metal block is added to the cart, the total mass increases by 500 grams as can be seen in the function table:

b	*f(b)*
0	5,500
1	6,000
2	6,500
3	7,000
4	7,500

So each block must have a mass of 500 grams, which is the coefficient of the *x*-term in the function.

Part B: When there are *no* metal blocks in the cart, $b = 0$. The only mass left is the mass of the cart; therefore, the cart has a mass of 5,500 grams.

Item 19

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: O1.a – Use properties of number systems within the set of real numbers to verify or refute conjectures or justify reasoning and to classify, order, and compare real numbers.

Correct Answer: D

Explanation of Distractors:

Rationale A	Thinks the number is irrational because of the exponents. Fails to recognize that 2^4 is a perfect square.
Rationale B	Confuses irrational numbers with perfect squares or fails to recognize that $900 = 30^2$ is a perfect square.
Rationale C	Thinks the number is irrational because of the decimal inside the radical. Fails to recognize that the product under the square root is a perfect square.

Instructional Implications: This item fundamentally relies on the knowledge that taking a square root means identifying pairs of factors that are the same and removing them from under the radical symbol. When the process of removing all paired factors results in no unpaired factors remaining under the radical, the number is rational. In addition, it is helpful if students recognize relatively small square numbers, often called *perfect squares,* even when they appear in exponential, decimal or factored form.

Sample Solution:

$$\sqrt{10(80)} = \sqrt{800} = \sqrt{2(400)} = 20\sqrt{2}$$

Item 20

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: O1.d – Use the properties of radicals to rewrite numerical expressions containing square roots in different but equivalent forms or to solve problems.

Correct Answer: C

Explanation of Distractors:

Rationale A	Multiplied integers together (2×4) and radicands $-(\sqrt{5} \times \sqrt{5})$ together, then combined $(8 - 5 = 3)$, but failed to multiply the whole numbers and radicals from the first and second binomials
Rationale B	Multiplied all pairs of terms except for the last term from each binomial, and then dropped the radical when the second and third terms were added. $\begin{aligned}(2-\sqrt{5})(4+\sqrt{5}) &\neq 8 + 2\sqrt{5} - 4\sqrt{5} \\ &\neq 8 - 2 \\ &= 6\end{aligned}$
Rationale D	Confused the sign on the product of the radicals. $\begin{aligned}(2-\sqrt{5})(4+\sqrt{5}) &\neq 8 + 2\sqrt{5} - 4\sqrt{5} + 5 \\ &\neq 13 - 2\sqrt{5}\end{aligned}$

Instructional Implications: The key to student success when multiplying the binomials is attention to the signs of the numbers and the multiplication of radicals.

Sample Solution:

$$\begin{aligned}(2-\sqrt{5})(4+\sqrt{5}) &= 8 + 2\sqrt{5} - 4\sqrt{5} - 5 \\ &= 3 - 2\sqrt{5}\end{aligned}$$

Item 21

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: D1.c – Evaluate the reliability of reports based on data published in the media.

Correct Answer: A

Explanation of Distractors:

Rationale B	Chooses a statement that is relatively true of the data, but vertical scale used on the graph makes SUV-5 appear to have much lower satisfaction ratings.

Rationale C	Chooses a statement that is relatively true of the data, but vertical scale on the graph used does not make the ratings appear equal.
Rationale D	Chooses a statement that is relatively true of the data, but vertical scale on the graph used does not make the ratings appear equal.

Instructional Implications: For success with this item, students need to pay attention to the effect of the choice of the vertical scale. Comparing the visual impact of this type of graph with different vertical scales would be helpful as students develop such insight.

Sample Solutions: The graph reports satisfaction ratings in tenths of a point, with a vertical scale ranging from 8.0 through 9.2. While all five SUVs have satisfaction between 8.1 and 9.2 on a 10-point scale, this choice of vertical scale makes vehicles with very similar data look like their data is dramatically different and makes SUV-1 look much better than the other SUVs. Students need to recognize that although the rating of the SUVs are similar, the impact of the visual representation of the graph might alter the impression of the data for the organization and their purposes stated in the item answer choices.

Item 22

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: N1.b – Analyze a table, numerical pattern, graph, equation or context to determine whether a linear, quadratic or exponential relationship could be represented. Or, given the type of relationship, determine elements of the table, numerical pattern or graph.

Correct Answer: D

Explanation of Distractors:

Rationale A	This function describes exponential growth rather than decay and has a y-intercept of $(0,1)$. It is possible that the y-intercept has been mistaken for the base.

Rationale B	Recognizes that the graph is decreasing but fails to recognize that $f(x) = -2^x = -(2^x)$ is negative for all values of x and would be graphed only in quadrants III and IV. Assumes that the decreasing nature of the function means the coefficient is negative and that the base is 2 because the y-intercept is 2.
Rationale C	Recognizes exponential decay but not the correct y-intercept.

Instructional Implications: Despite the fact that all choices for this item involve exponential equations, one goal of the underlying benchmark is that students distinguish among linear, quadratic and exponential functions whether they are represented graphically, symbolically or in a contextual situation. For exponential functions, students will also need to distinguish between growth and decay functions. Graphically this is fairly straightforward, but symbolically it can be quite confusing. Students need to recognize that $f(x) = 2^{-x}$ is the same function as $f(x) = \left(\frac{1}{2}\right)^x$ and $f(x) = -2^x$ is the reflection of $f(x) = 2^x$, a growth function, reflected over the x-axis. Helping students identify and interpret these nuances correctly is at the core of this item.

Sample Solution: This is a graph of an exponential decay function. There are two key pieces to finding the equation of this graph. First, the function has its y-intercept at 2. In order for the general exponential function $f(x) = a \cdot b^x$, where $b > 0$ and $b \neq 1$, to be equal to 2 when $x = 0$, the coefficient a must be 2 because any non-zero real number to the power of 0 equals 1. Secondly, to be a decay function, the value of b is restricted to $0 < b < 1$ or the exponent must be $-x$. Answer choice D is the only option that satisfies both of these conditions.

Item 23

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: L2.b – Solve equations involving the absolute value of a linear expression.

Correct Answer: D

Explanation of Distractors:

Rationale A	Did not realize there was no solution and solved the equation as if it did not contain an absolute value. $3x + 2 = -1$ $3x = -3$ $x = -1$
Rationale B	Solved the equation as if it did not contain an absolute value, then changed the sign of x, mistakenly thinking that the result of any problem involving an absolute value must not be negative. $3x + 2 = -1$ $3x = -3$ $x = 1 \rightarrow x = 1$
Rationale C	Did not realize there was no solution and solved the equation by setting the absolute value of the expression to 1 and -1, failing to check either answer obtained. $3x + 2 = -1$ or $3x + 2 = 1$ $3x = -3$ or $3x = -1$ $x = -1$ or $x = -\frac{1}{3}$

Instructional Implications: It is important for students to understand the definition of absolute value, which states: For all real numbers, x:

$$|x| = x, \text{for } x \geq 0$$
$$|x| = -x, \text{for } x < 0.$$

When students analyze an equation before beginning work to solve it, they may be able to avoid unnecessary work if they are able to make connections between definitions and what is given in the problem.

Sample Solutions:

Observation Approach: Looking at this item as an absolute value item, instead of just an equation to solve, will help students realize that an absolute value cannot equal a negative number. Therefore, by the above definition, there is no linear expression whose absolute value equals a negative value, and so there is no solution to this equation.

Algebraic Approach: $|3x+2|=-1$

If $3x+2 \geq 0$,
then $|3x+2| = 3x+2$
so $3x+2=-1$
$3x=-3$
$x=-1$
However, when $x=-1$ is substituted into the original equation, $|3(-1)+2|=-1$, the result, $|-1|=-1$ is false.

If $3x+2<0$,
then $|3x+2| = -(3x+2)$
so $-3x-2=-1$
$-3x=1$
$x=-\frac{1}{3}$
However, when $x=-\frac{1}{3}$ is substituted into the original equation, $\left|3\left(-\frac{1}{3}\right)+2\right|=-1$, the result, $|1|=-1$ is false.

In both cases, the potential solution does not satisfy the required initial condition so there is no solution to the equation.

Item 24

Item Type: Short Answer

Calculator: NOT Permitted

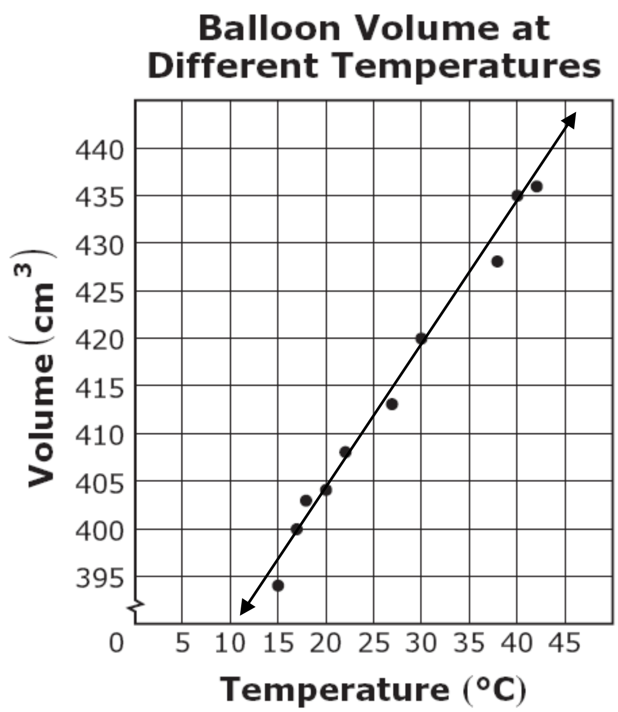
Benchmark: D1.a – Interpret and compare linear models for data that exhibit a linear trend including contextual problems.

Correct Answer: Any value between 422 and 428 and an explanation that justifies the answer.

Instructional Implications: To make the prediction required for this item, it will be helpful for students to sketch a linear trend line for data. The goal when sketching this line should be to have some data points above the line and some below in a reasonably random way. The difference between the *y*-coordinate of each point and the *y*-coordinate of the point on the line having the same *x*-coordinate is called the *residual*; the goal is for the residuals to be randomly distributed about zero, some being positive and some negative. Once the trend line is drawn, students should estimate the *y*-coordinate for an *x*-coordinate of 35 directly or find some way to interpolate the value using points whose *y*-coordinates are relatively easy to estimate.

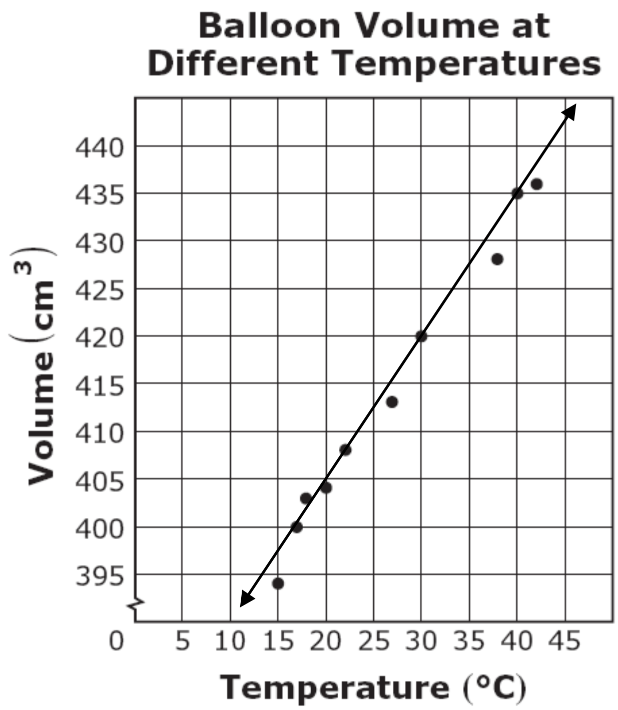
Sample Solutions:

Single Point Approach: Draw a line that approximates the trend in these data.



The *y*-value for the point where the line crosses the vertical line where the temperature is 35°C appears to be slightly above the horizontal line where the volume is 425 cm^3, and so the volume might be predicted to be 427 cm^3.

Interpolation Approach: Draw a line that approximates the trend in these data.



The line appears to pass through the points $(30, 420)$ and $(40, 435)$. Averaging the *x*- and *y*-coordinates will give the midpoint of the line segment, which will yield the desired *x*-coordinate, 35. The midpoint value for the *y*-coordinate, or volume, will be approximately 427.5.

$$(x, y)$$
$$\left(\frac{30 + 40}{2}, \frac{420 + 435}{2}\right)$$
$$(35, 427.5)$$

Item 25

<u>Item Type</u>: Multiple Choice

<u>Calculator</u>: Permitted

<u>Benchmark</u>: N2.a – Solve equations involving several variables for one variable in terms of the others.

<u>Correct Answer</u>: D

Explanation of Distractors:

Rationale A	After solving for L, reduces denominator with only one term in numerator. $S = \pi r L + \pi r^2$ $\dfrac{S - \pi r^2}{\pi r} = L$ $L \neq S - r$
Rationale B	After incorrectly solving for L, reduces denominator with only one term in numerator. $S = \pi r L + \pi r^2$ $-\pi r L \neq S - \pi r^2$ $L \neq -\dfrac{S - \pi r^2}{\pi r}$ $L \neq r - S$
Rationale C	Makes mistake with a sign. $S = \pi r L + \pi r^2$ $-\pi r L \neq S - \pi r^2$ $L \neq -\dfrac{S - \pi r^2}{\pi r}$ $L \neq \dfrac{\pi r^2 - S}{\pi r}$

Instructional Implications: Since L appears in only one term, the goal is to isolate that term on one side of the equation and divide by the coefficient. Students must take care reducing the resulting fraction.

Sample Solution:

$$S = \pi r L + \pi r^2$$
$$S - \pi r^2 = \pi r L$$
$$L = \frac{S - \pi r^2}{\pi r}$$

Item 26

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: D2.a – Use counting principles to determine the number of ways an event can occur. Interpret and justify solutions.

Correct Answer: D

Explanation of Distractors:

Rationale A	One appetizer, one entrée and one dessert: $1+1+1$
Rationale B	Thinks one of each and uses addition: $7+5+3$
Rationale C	Neglects to include dessert: 7×5

Instructional Implications: Students' prior experience with tree diagrams will be helpful for success with this item. By the time algebra is studied, students should have realized that this situation calls for multiplication of possibilities and should no longer need to rely on the actual diagram.

Sample Solutions: For each of 5 appetizers, 7 different entrees are possible for a total of $7\times 5=35$ different possibilities. For each of these 35 possibilities, 3 different desserts are possible for a total of $5\times 7\times 3=35\times 3=105$ different possible meals.

Item 27

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: L2.a – Solve single-variable linear equations and inequalities with rational coefficients.

Correct Answer: A

Explanation of Distractors:

Rationale B	Fails to account for the change in the direction of the inequality when dividing by –4. $\frac{1}{2}-\frac{2}{3}x<\frac{5}{6}$ $\left(\frac{1}{2}-\frac{2}{3}x<\frac{5}{6}\right)6$ $3-4x<5$ $-4x<2$ $x\not<-\frac{1}{2}$
Rationale C	Incorrectly subtracts and fails to account for the change in the direction of the inequality when dividing by –4. $\frac{1}{2}-\frac{2}{3}x<\frac{5}{6}$ $\left(\frac{1}{2}-\frac{2}{3}x<\frac{5}{6}\right)6$ $3-4x<5$ $-4x\not<8$ $x<-2$
Rationale D	Incorrectly subtracts but divides by –4 correctly. $\frac{1}{2}-\frac{2}{3}x<\frac{5}{6}$ $\left(\frac{1}{2}-\frac{2}{3}x<\frac{5}{6}\right)6$ $3-4x<5$ $-4x\not<8$ $x>-2$

Instructional Implications: Clearing this inequality of fractions is a helpful first step although students may certainly find the solution without doing so. Care must be taken to reverse the inequality sign when multiplying or dividing by a negative number.

Sample Solution:

$$\frac{1}{2}-\frac{2}{3}x<\frac{5}{6}$$
$$\left(\frac{1}{2}-\frac{2}{3}x<\frac{5}{6}\right)6$$
$$3-4x<5$$
$$-4x<2$$
$$x>-\frac{1}{2}$$

Item 28

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: O2.c – Factor simple polynomial expressions with or without a context.

Correct Answer: A

Explanation of Distractors:

Rationale B	$2x^3+8x^2-24x=2x\left(x^2+4x-12\right)$ $\neq 2\text{x}(\text{x - }6)(x+2)$
Rationale C	$2x^3+8x^2-24x=2x\left(x^2+4x-12\right)$ $\neq 2\text{x}(\text{x}+4)(x-3)$
Rationale D	$2x^3+8x^2-24x=2x\left(x^2+4x-12\right)$ $\neq 2\text{x}(\text{x - }4)(x+3)$

Instructional Implications: This problem is easier to approach if students recognize that there is a common factor in all terms before attempting to factor the polynomial. Once the common factor has been removed, care must be taken in factoring the remaining quadratic expression, paying special attention to the signs of the numbers.

Sample Solution:

$$2x^3 + 8x^2 - 24x = 2x(x^2 + 4x - 12)$$
$$= 2x(x + 6)(x - 2)$$

Item 29

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: L2.e – Recognize, express and solve problems that can be modeled using single-variable linear equations; one- or two-variable inequalities; or two-variable systems of linear equations. Interpret their solutions in terms of the context of the problem.

Correct Answer: B

Explanation of Distractors:

Rationale A	When eliminating the *d* term of the second equation, only the *d* term is multiplied by -2. $\begin{array}{r} 2d + 275m = 140.75 \\ \underline{-2d - 95m \neq 59.75} \\ 180m \neq 81 \\ m \neq 0.45 \end{array}$ $\quad$ $\begin{array}{r} d + (95)(0.45) = 59.75 \\ d + 42.75 = 59.75 \\ d = 17 \end{array}$
Rationale C	When eliminating the *d* term of the second equation, the second equation is not multiplied by a constant, instead the *d* terms just disappear. $\begin{array}{r} 2d + 275m = 140.75 \\ \underline{-d + 95m = 59.75} \\ 370m \neq 200.5 \\ m \not\approx 0.54 \end{array}$ $\quad$ $\begin{array}{r} d + (95)(0.54) \neq 59.75 \\ d + 51.30 \neq 59.75 \\ d \neq 8.45 \end{array}$
Rationale D	Only the first trip is used to solve for the number of miles, and only the second trip is used to solve for the number of miles. $95m = 59.75$ $\quad$ $2d = 140.75$ $m = 0.628 \approx 0.63$ $\quad$ $d = 70.375 \approx 70.38$

Instructional Implications: Care must be taken to notice that the first rental is for two days while the second is for only one day. While not part of this problem, students should be encouraged to clearly define the variables they are using in any situation for which they must write equations. Once the system of equations is solved, students should be encouraged to develop the habit of clearly stating the result using appropriate units and in terms of the context of the problem.

Sample Solution: Let d = the cost of the car per day
Let m = the cost of the car per mile driven

On the first trip, the car was rented for 2 days and driven 275 miles and the total cost was $140.75 so the situation can be represented by the equation $2d + 275m = 140.75$.

On the second trip, the car was rented for only 1 day and driven only 95 miles for a total cost of $59.75 so this situation can be represented by the equation $d + 95m = 59.75$.

Solving these two equations simultaneously yields:

$$\begin{aligned} 2d + 275m &= 140.75 \\ d + 95m &= 59.75 \end{aligned}$$

$$\begin{aligned} 2d + 275m &= 140.75 \\ -\,2\left(d + 95m = 59.75\right) \end{aligned}$$

$$\begin{aligned} 2d + 275m &= 140.75 \\ -\,2d - 190m &= -119.50 \\ 85m &= 21.25 \\ m &= 0.25 \end{aligned}$$

$$\begin{aligned} 2d + 275(0.25) &= 140.75 \\ 2d + 68.75 &= 140.75 \\ 2d &= 72 \\ d &= 36 \end{aligned}$$

The car rents for $36 per day and each mile driven costs $0.25.

Item 30

Item Type: Short Answer

Calculator: Permitted

Benchmark: O2.b – Add, subtract and multiply polynomial expressions with or without a context.

Correct Answer: Full credit requires work or explanation of a strategy that leads to the expression $x^3 - x^2 - 4x + 14$.

Instructional Implications: For success with this item, students must understand the application of the distributive property of multiplication over addition, paying special attention to the signs.

Sample Solution:

$$x^2(x+2) - 3x(x+2) + 2(x+7)$$
$$x^3 + 2x^2 - 3x^2 - 6x + 2x + 14$$
$$x^3 - x^2 - 4x + 14$$

Item 31

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: N1.c – Recognize and solve problems that can be modeled using a quadratic function. Interpret the solution in terms of the context of the original problem.

Correct Answer: D

Explanation of Distractors:

Rationale A	Squares the binomials incorrectly $(x+2)^2 + (x+6)^2 = (2x)^2$ $x^2 + 4 + x^2 + 36 \neq 4x^2$ $-2x^2 + 40 = 0$

	$x = \dfrac{\pm\sqrt{-4(-2)(40)}}{2(-2)}$ $= \dfrac{\pm\sqrt{320}}{-4}$ $= \dfrac{\pm 8\sqrt{5}}{-4}$ $= \pm 2\sqrt{5}$ Rejects the negative value, uses $x = 2\sqrt{5}$ and finds the side lengths of $2\sqrt{5}+2$, $2\sqrt{5}+6$, and $2(2\sqrt{5}) = 4\sqrt{5}$, finds the perimeter of $2\sqrt{5}+2+2\sqrt{5}+6+4\sqrt{5} = 8+8\sqrt{5} \approx 26$
Rationale B	Finds x correctly but incorrectly uses the value of x. $(x+2)^2 + (x+6)^2 = (2x)^2$ $x^2 + 4x + 4 + x^2 + 12x + 36 = 4x^2$ $-2x^2 + 16x + 40 = 0$ $x = \dfrac{-16 \pm \sqrt{(16)^2 - 4(-2)(40)}}{2(-2)}$ $= \dfrac{-16 \pm \sqrt{256+320}}{-4}$ $= \dfrac{-16 \pm 24}{-4}$ $= -2, \quad 10$ Rejects the negative value, multiplies 10 by 3 to get the perimeter instead of substituting to compute the lengths of each side and then adds the side lengths together.
Rationale C	Determines the correct value for x, but adds 12, 16 and 10, instead of 12, 16 and 20.

Instructional Implications: Success with this item requires care and thoughtfulness in carrying out a multi-step process. Having students articulate a plan for solving the problem may help focus their efforts and to stay on track throughout the process. Students must identify the need to determine a value for x, using the Pythagorean Theorem and either the quadratic equation or factoring. Then one potential x-value must be rejected because it would yield negative values for the length of a side of the triangle. The acceptable *x*-value must be substituted so that the lengths of the three sides can be determined. Finally the lengths added to determine the perimeter. Other key concepts include squaring binomials and use of the quadratic formula or factoring as part of finding the x-value.

Sample Solutions:

Quadratic Formula Approach:

$$
\begin{aligned}
(x+2)^2 + (x+6)^2 &= (2x)^2 \\
x^2 + 4x + 4 + x^2 + 12x + 36 &= 4x^2 \\
-2x^2 + 16x + 40 &= 0
\end{aligned}
$$

$$
\begin{aligned}
x &= \frac{-16 \pm \sqrt{(16)^2 - 4(-2)(40)}}{2(-2)} \\
&= \frac{-16 \pm \sqrt{256 + 320}}{-4} \\
&= \frac{-16 \pm 24}{-4} \\
&= -2, \quad 10
\end{aligned}
$$

The value of $x = -2$ is rejected because it is a negative value. Therefore, the sides of the triangle have lengths:

$10 + 2 = 12$ $\qquad$ $10 + 6 = 16$ $\qquad$ $2(10) = 20$

The perimeter is $12 + 16 + 20 = 48$ units.

Factoring Approach:

$$
\begin{aligned}
(x+2)^2 + (x+6)^2 &= (2x)^2 \\
x^2 + 4x + 4 + x^2 + 12x + 36 &= 4x^2 \\
-2x^2 + 16x + 40 &= 0 \\
-2(x^2 - 8x - 20) &= 0 \\
-2(x+2)(x-10) &= 0 \\
x &= -2, \ 10
\end{aligned}
$$

The value of $x = -2$ is rejected because it is a negative value. Therefore, the sides of the triangle have lengths:

$10 + 2 = 12$ $\qquad$ $10 + 6 = 16$ $\qquad$ $2(10) = 20$

The perimeter is $12 + 16 + 20 = 48$ units.

Item 32

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: O2.d – Use the properties of radicals to convert algebraic expressions containing square roots into different but equivalent forms or to solve problems.

Correct Answer: B

Explanation of Distractors:

Rationale A	Confused exponents when taking the square roots. $\sqrt{y^5z^{10}} = \sqrt{(y^2)^2 \cdot y \cdot (z^5)^2} \neq y^2\lvert z^2\rvert\sqrt{y^2z^5}$
Rationale C	Did not include the absolute value sign.
Rationale D	Did not take the square root of y^5.

Instructional Implications: Students will need to know how to simplify square roots of numerical quantities and translate that knowledge into this situation with variables having exponents. After the simplification process, students must remember to use the absolute value of the variable, if necessary. The absolute value is a necessary part of the answer for variables with an even exponent that, when the square root is taken, have an odd exponent. The expression $\sqrt{y^5z^{10}}$ represents only the principal or positive square root – i.e., the non-negative value that when squared gives the product under the radical. It is not known whether the variable is positive or negative when the exponent of the variable under the radical is even, so using the absolute value for that variable ensures that the result is the principal square root of the expression.

Sample Solution: $\sqrt{y^5z^{10}} = \sqrt{(y^2)^2 \cdot y \cdot (z^5)^2} = y^2\lvert z^5\rvert\sqrt{y}$ or $y^2\lvert z\rvert^5\sqrt{y}$

Item 33

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: D2.b – Apply probability concepts to determine the likelihood an event will occur in practical situations.

Correct Answer: D

Explanation of Distractors:

Rationale A	Determines the percentage of students, $\frac{38}{50} = 0.76$ or 76%, but fails to determine the *number* of students out of 652.
Rationale B	Finds the percentages incorrectly. $\frac{38}{2} = 19$ $0.19(652) = 123.88$ or about 123 students.
Rationale C	Incorrectly uses the 38 as a percentage. $0.38 \times 652 = 247.76$ or about 247 students.

Instructional Implications: This item offers the opportunity to discuss the assumptions inherent in making predictions about a population from a sample. In this case, the sample is said to be random. Some samples should also be stratified in some way to better represent the population. However, from the ADP Algebra I End-of-Course Exam Expectations of Knowledge, all samples in the exam are considered to be representative of the population. Students should understand that even if all care is taken in drawing the sample, the number 495 is only an *estimate* of the number of breakfast-eating students in the school.

Sample Solution: The percentage of students in the sample that eat breakfast was:

$$\frac{38}{50} = 0.76 .$$

If this sample is used to predict the number of students in the whole population who eat breakfast, that percentage should be multiplied by the number of students in the population, $0.76(652) = 495.52 \approx 495$. Therefore, the prediction is that 495 students eat breakfast every morning.

Item 34

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: D1.b – Use measures of center and spread to compare and analyze data sets.

Correct Answer: A

Explanation of Distractors:

Rationale B	Miscalculates the mean and median.
Rationale C	Miscalculates the median. Uses an incorrect definition of median – uses the middle number in the unordered list; the original median is then 324 and the new median is 284.
Rationale D	Miscalculates the mean. Fails to include the second 244 value in calculation.

Instructional Implications: While both the original mean and median and the mean and median of the set with 486 added can all be calculated and compared, it would be much more efficient if students could think conceptually about this problem.

Sample Solutions:

Conceptual approach: Four of the original numbers are less than 486, and the original median is a unique member of the set of numbers, 486 will increase the median. The mean of the original set of numbers is 363.2; since $486 > 363.2$, adding that number to the set will also increase the mean.

Algebraic Approach: The mean of the original set of numbers is:

$$\frac{691+313+324+244+244}{5} = 363.2.$$

The addition of 486 would make the new mean:

$$\frac{691+313+324+244+244+486}{6} = 383.\overline{6}.$$

So, the mean will increase.

The original median is 313, the middle value of the five numbers when this numbers are listed in order. Since $486 > 313$ the new median will be:

$$\frac{313+324}{2} = 318.5 .$$

So, the median will also increase.

Item 35

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: L1.d – Recognize, express and solve problems that can be modeled using linear functions. Interpret their solutions in terms of the context of the problem.

Correct Answer: D

Explanation of Distractors:

Rationale A	Multiplies the number of classes by 3.
Rationale B	Disregards the 3 free classes.
Rationale C	Switches the 10 and 3.

Instructional Implications: Students need practice translating verbal situations into symbolic expressions. In this problem, care must be taken to properly account for the three free classes.

Sample Solution: If x represents the number of classes taken next month, then x - 3 represents the number of classes that must be paid for during the special that month. The total cost, $C(x)$, is the sum of the membership fee plus the cost of the classes:

$$C(x) = 10(x-3) + 52 .$$

Item 36

Item Type: Extended Response

Calculator: Permitted

Benchmark: Extended-response items can be written to address multiple aspects of the standard. This particular item was written to the L standard, *Linear Relationships*, and mainly addresses the following benchmarks within the standard.

L.1.a – Recognize, describe and represent linear relationships using words, tables, numerical patterns, graphs and equations. Translate among these representations.
L.1. d – Recognize, express and solve problems that can be modeled using linear functions. Interpret their solutions in terms of the context of the problem.
L.2.e – Recognize, express and solve problems that can be modeled using single-variable linear equations; one- or two-variable inequalities; or two-variable systems of linear equations. Interpret their solutions in terms of the context of the problem.

Correct Answers:

Part A: $N = 1.19E$. To earn full credit, student solutions must include a reasonable written explanation and/or algebraic work that justifies the answer.

Part B: 235.3 pounds. To earn full credit, student solutions must include a reasonable written explanation and/or algebraic work that justifies the answer.

Part C: 2.38 pounds. To earn full credit, student solutions must explain the reasoning behind the answer.

Instructional Implications: Students should be encouraged to define the variables they use to write any contextual equation, with special attention to which variable represents the independent and dependent variable. Reversing the independent and dependent variables can cause the equation to be incorrect, therefore resulting in errors in the remainder of the problem. In this case, the point-slope form of the line is probably the most efficient form to use for Part A. Part B requires only correct substitution and solving the resulting equation. Part C can be approached by comparing the values of N when the weight on Earth is E pounds and when it is $E+2$ pounds.

<u>Sample Solutions</u>:

Part A:
Let E = the weight (in pounds) of an object on Earth (independent)
Let N = the weight (in pounds) of an object on Neptune (dependent)
Then the equation of the line represents the relationship between these two quantities.

Using the coordinates of the points indicated on the graph, the slope of the line is given by:

$$m = \frac{\Delta N}{\Delta E} = \frac{214.2 - 47.6}{180 - 40} = 1.19 .$$

Its y-intercept appears to be 0, but to check this, the equation of the line can be found using this slope and the point $(40, 47.6)$:

$$\begin{aligned} N - 47.6 &= 1.19(E - 40) \\ N &= 1.19E - 47.6 + 47.6 . \\ N &= 1.19E \end{aligned}$$

Therefore, the equation of the line is $N = 1.19E$.

Part B: Substituting 280 pounds for N:

$$\begin{aligned} 280 &= 1.19E \\ E &= 235.2941176.... \\ E &\approx 235.3 \end{aligned}$$

Part C: The original weight on Neptune for an object weighing E pounds on Earth is represented by $N_1 = 1.19E$. If E is increased by 2 pounds, then the new weight on Neptune is represented by:

$$\begin{aligned} N_2 &= 1.19(E+2) \\ &= 1.19E + 2.38 \end{aligned} .$$

The difference between the two weights is:

$$\begin{aligned} N_2 - N_1 &= 1.19E + 2.38 - 1.19E \\ &= 2.38 \text{ pounds} \end{aligned} .$$

Item 37

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: N1.c – Recognize and solve problems that can be modeled using a quadratic function. Interpret the solution in terms of the context of the original problem.

Correct Answer: D

Explanation of Distractors:

Rationale A	Uses an incorrect order of operations. $h(0) = 9 - 16(0)^2 = -7(0)^2 = 0$
Rationale B	Sets the height equal to zero. Finds the time in the air. $9 - 16t^2 = 0$ $(3 + 4t)(3 - 4t) = 0$ $t = \pm\frac{3}{4}$ and rejects the negative value.
Rationale C	Sets the height equal to zero. Solves incorrectly. $9 - 16t^2 = 0$ $(3 + 4t)(3 - 4t) = 0$ $t = \pm\frac{4}{3}$ and rejects the negative value.

Instructional Implications: This problem relies on an understanding that the initial height is that at which the ball starts and occurs at t = 0.

Sample Solution: When the time, t (in seconds), is set equal to zero, the associated height, h (in feet), will be the initial height from which the ball was dropped.

$$\begin{aligned} h(0) &= 9 - 16(0)^2 \\ &= 9 \text{ feet} \end{aligned}$$

Item 38

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: O2.a – Apply the laws of exponents to algebraic expressions with integral exponents to rewrite them in different but equivalent forms or to solve problems.

Correct Answer: C

Explanation of Distractors:

Rationale A	Adds exponents. $\frac{\left(x^{-2}y^{10}\right)^2}{x^5y^{-3}} = \frac{x^{-4}y^{20}}{x^5y^{-3}} \neq \frac{y^{17}}{x}$
Rationale B	Adds exponents in the numerator when squaring. $\frac{\left(x^{-2}y^{10}\right)^2}{x^5y^{-3}} \neq \frac{y^{12}}{x^5y^{-3}} \neq \frac{y^{15}}{x^5}$
Rationale D	Combines variables in the numerator and the denominator before squaring the variables in the numerator. $\frac{\left(x^{-2}y^{10}\right)^2}{x^5y^{-3}} \neq \frac{\left(y^{13}\right)^2}{x^7} \neq \frac{y^{26}}{x^7}$

Instructional Implications: For success on this item, it is important for students to know the order of operations. Students need to recognize that squaring the numerator must be completed before the consideration of dividing. Care must be taken with negative exponents, since this can be a problem area for some students.

Sample Solution: $\frac{\left(x^{-2}y^{10}\right)^2}{x^5y^{-3}} = \frac{x^{-4}y^{20}}{x^5y^{-3}} = \frac{y^{20-(-3)}}{x^{5-(-4)}} = \frac{y^{23}}{x^9}$

Item 39

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: N2.a – Solve equations involving several variables for one variable in terms of the others.

Correct Answer: C

Explanation of Distractors:

Rationale A	Incorrectly solves the equation, combines the y terms in the numerator and denominator by subtracting them. $\frac{4x+y}{3y}=2$ $\frac{4x}{2y}\neq 2$ $4x \neq 4y$ $x \neq y$
Rationale B	Incorrectly solves the equation, subtracts y from one side but adds y to the other side of the equation. $\frac{4x+y}{3y}=2$ $4x+y=6y$ $4x \neq 7y$ $\frac{4x}{7}\neq y$
Rationale D	Incorrectly solves the equation, multiplies incorrectly by $\left(\frac{1}{4x}\right)$ and by 3. $\frac{4x+y}{3y}=2$ $\left(\frac{1}{4x}\right)\left(\frac{4x+y}{3y}\right)=(2)\left(\frac{1}{4x}\right)$ $\frac{y}{3y}\neq\frac{2}{4x}$ $(3)\left(\frac{y}{3y}\right)\neq\left(\frac{2}{4x}\right)(3)$ $y\neq\frac{6}{4x}$ $y\neq\frac{3}{2x}$

Instructional Implications: The goal in this problem is to isolate the *y* variable on one side of the equation. To accomplish this, the first step should be to multiply both sides by the denominator. Often, this is the trickiest step for students.

Sample Solution:

$$\frac{4x+y}{3y}=2$$

$$(3y)\left(\frac{4x+y}{3y}\right)=(2)(3y)$$

$$4x+y=6y$$

$$4x=5y$$

$$\frac{4x}{5}=y$$

Item 40

Item Type: Multiple Choice

Calculator: NOT Permitted

Benchmark: L2.b – Solve equations involving the absolute value of a linear expression

Correct Answer: B

Explanation of Distractors:

Rationale A	Writes two equations, only one is correct. $5x-4=19$ or $-5x-4=19$ $5x=23$ or $-5x=23$ $x=\frac{23}{5}$ or $x=-\frac{23}{5}$
Rationale C	Fails to divide each term by 5. $5x-4=\pm 19$ $x-4=\pm\frac{19}{5}$ $x=-\frac{1}{5},\ \frac{39}{5}$

Rationale D	Writes two equation, only one is correct $5x - 4 = 19$ $5x + 4 = 19$ $5x = 23$ or $5x = 15$ $x = \frac{23}{5}$ $x = 3$

Instructional Implications: The definition of absolute value is the underlying concept necessary for success in solving this problem. The definition states: For all real numbers, x:

$$|x| = x, \text{for } x \geq 0$$
$$|x| = -x, \text{for } x < 0$$

Once students have correctly stated the two cases, solving the resulting linear equation is straightforward.

Sample Solution: Using the definition of absolute value:

If $5x - 4 \geq 0$, then

$$\begin{aligned} |5x - 4| &= 5x - 4 \\ 5x - 4 &= 19 \\ 5x &= 23 \\ x &= \frac{23}{5} \end{aligned}$$

If $5x - 4 < 0$, then

$$\begin{aligned} |5x - 4| &= -(5x - 4) \\ -(5x - 4) &= 19 \\ 5x - 4 &= -19 \\ 5x &= -15 \\ x &= -3 \end{aligned}$$

Both of these answers meet the conditions of their respective cases and when substituted into the original equation, both solutions yield true statements. Therefore, both are solutions for the problem.

Item 41

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: D2.a – Use counting principles to determine the number of ways an event can occur. Interpret and justify solutions.

Correct Answer: B

Explanation of Distractors:

Rationale A	Expresses the number of speed settings.
Rationale C	Computes the sum of the options.
Rationale D	Confuses the counting principle of addition with the fundamental counting principle.

Instructional Implications: Careful reading is the key to success with this item. Students need to use the statement "There are 3 times as many speed settings as file settings" to express the number of speed settings. They also need to correctly interpret the statement "the number of combinations." The vocabulary itself contains a hint – in mathematics, the term "combination" is the number of ways independent events can be put together in situations where order does not matter. Even if students are not familiar with the term, "combinations" should motivate a tree diagram sort of approach from earlier school experience.

Sample Solution: Let x = the number of possible file settings.
Then $3x$ = the number of possible speed settings.

The number of combinations – the number of different ways that speed settings can be paired with file settings – is given by
$3x \cdot x = 3x^2$.

Item 42

Item Type: Short Answer

Calculator: Permitted

Benchmark: N2.b – Solve single-variable quadratic equations.

Correct Answer: $x = 1$ and $x = -7$. To earn full credit, student solutions must include a reasonable written explanation and/or algebraic work that justifies the answer.

Instructional Implications: Beginning this problem by substituting 7 for y will simplify the process regardless of the approach used to solve the equation. Students should have multiple approaches at their disposal when solving a quadratic. Students should analyze the quadratic before beginning to see which method is best. Although factoring and the quadratic equation both yield the same answer, the quadratic equation introduces more places for students to make a computation error in this item.

Sample Solutions:

Quadratic Equation Approach	Expansion/Factoring Approach
$(x+3)^2+(y-4)^2=5^2$	$(x+3)^2+(y-4)^2=5^2$
$(x+3)^2+(7-4)^2=25$	$(x+3)^2+(7-4)^2=25$
$x^2+6x+9+9=25$	$x^2+6x+9+9=25$
$x^2+6x-7=0$	$x^2+6x-7=0$
	$(x-1)(x+7)=0$
$x=\dfrac{-6\pm\sqrt{(6)^2-4(1)(-7)}}{2(1)}$	$x-1=0$ or $x+7=0$
$=\dfrac{-6\pm\sqrt{36+28}}{2}$	$x=1$ $x=-7$
$=\dfrac{-6\pm\sqrt{64}}{2}$	
$=1,\ -7$	

In either case, $x=-1$ or $x=7$.

Item 43

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: O1.b – Use rates, ratios and proportions to solve problems, including measurement problems.

Correct Answer: B

Explanation of Distractors:

Rationale A	Uses the incorrect conversion factor for hours to seconds. $\dfrac{42\text{ mi}}{1\text{ hr}}\cdot\dfrac{1\text{ hr}}{3{,}600\text{ sec}}\cdot\dfrac{5{,}400\text{ ft}}{1\text{ mi}}=63.0\dfrac{\text{ft}}{\text{sec}}$

Rationale C	Uses the incorrect conversion for miles to feet. $\frac{42\text{ mi}}{1\text{ hr}} \cdot \frac{1\text{ hr}}{60\text{ min}} \cdot \frac{1\text{ min}}{60\text{ sec}} \cdot \frac{5{,}000\text{ ft}}{1\text{ mi}} = 58.3\frac{\text{ft}}{\text{sec}}$
Rationale D	Uses reciprocals of conversion fractions, ignores units and simply inserts them at the end. $\frac{42\text{ mi}}{1\text{ hr}} \cdot \frac{60}{1} \cdot \frac{60}{1}\frac{1}{5{,}280} = 28.6\frac{\text{ft}}{\text{sec}}$

Instructional Implications: Since the use of a calculator is permitted for this item, the burden of the problem lies in the knowledge of the conversion factors and multiplying by the correct ratio. Attention to units, sometimes called *dimensional analysis* or *unit analysis* is helpful to be sure that the ratio used is not the reciprocal of the one that is needed.

Sample Solution:

$$\frac{42\text{ mi}}{1\text{ hr}} \cdot \frac{1\text{ hr}}{60\text{ min}} \cdot \frac{1\text{ min}}{60\text{ sec}} \cdot \frac{5{,}280\text{ ft}}{1\text{ mi}} = 61.6\frac{\text{ft}}{\text{sec}}$$

Item 44

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: L1.b – Describe, analyze and use key characteristics of linear functions and their graphs.

Correct Answer: C

Explanation of Distractors:

Rationale A	Uses the *y*-intercept value as the slope value and determines that they are the same.
Rationale B	Incorrectly concludes that the equations are multiples of each other.
Rationale D	Notices that the slopes are different but fails to notice that they are negative reciprocals.

Instructional Implications: Success on this item relies primarily on knowledge of the implication for slope when comparing two or more lines in the plane. Students should have practice with problems that require them to determine the slope of a linear equation. They will also benefit from practice that requires them to go the extra step to consider whether intersecting lines might be perpendicular. In this item, the four choices serve as a reminder for students, but that may not always be the case.

<u>Sample Solutions</u>:

<u>Slope-intercept Approach</u>:

$$\begin{aligned} -6x + 15y &= 5 \\ 15y &= 5 + 6x \\ y &= \frac{5}{15} + \frac{6}{15}x \\ y &= \frac{2}{5}x + \frac{1}{3} \end{aligned} \qquad \text{and} \qquad \begin{aligned} 30x + 12y &= 4 \\ 12y &= 4 - 30x \\ y &= \frac{4}{12} - \frac{30}{12}x \\ y &= -\frac{5}{2}x + \frac{1}{3} \end{aligned}$$

Because these two lines have different slopes, they must intersect. In fact, the slopes of the lines are negative reciprocals of each other and such lines are perpendicular to each other.

<u>Ratio of Coefficients Approach</u>: The slope of a linear equation of the form $Ax + By = C$, is given by the ratio $-\frac{A}{B}$.

$$\text{For } -6x + 15y = 5, \quad \begin{aligned} m &= -\frac{A}{B} \\ &= -\frac{-6}{15} \\ &= \frac{2}{5} \end{aligned} \qquad \text{For } 30x + 12y = 4, \quad \begin{aligned} m &= -\frac{A}{B} \\ &= -\frac{30}{12} \\ &= -\frac{5}{2} \end{aligned}$$

Because these two lines have different slopes, they must intersect. In fact, the slopes of the lines are negative reciprocals of each other, and such lines are perpendicular to each other.

Item 45

<u>Item Type</u>: Multiple Choice

Calculator: Permitted

Benchmark: L2.c – Graph and analyze the graph of the solution set of a two-variable linear inequality.

Correct Answer: B

Explanation of Distractors:

Rationale A	Solves for “greater than or equal to” instead of just “greater than.”
Rationale C	Solves for “less than or equal to” instead of just “less than.”
Rationale D	Writes the result in the incorrect order without attention to the inequality sign.

Instructional Implications: To solve this problem, students must first understand what it means for a point to be “part of the solution” of the linear inequality. That point must make the inequality statement true. Identifying the family of linear functions that correspond to this inequality will also help students visualize what *y*-intercept would be required.

Sample Solution: For the origin to be part of the solution of the inequality coordinates of the point $(0, 0)$ must satisfy the inequality.

$$6x + y < p$$
$$x = 0 \text{ and } y = 0$$
$$6 \cdot 0 + 0 < p$$
$$0 < p$$
$$p > 0$$

Item 46

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: O2.b – Add, subtract and multiply polynomial expressions with or without a context.

Correct Answer: D

Explanation of Distractors:

Rationale A	Did not distribute the negative sign over all the terms in the second polynomial – applies to first term only. $\left(a^3 - 5a + b - 2\right) - \left(3a^3 + 5a - b + 2\right) \neq$ $a^3 - 5a + b - 2 - 3a^3 + 5a - b + 2 \neq$ $-2a^3$
Rationale B	Did not distribute the negative sign correctly over all the terms in the second polynomial – applies to the first and third terms only. $\left(a^3 - 5a + b - 2\right) - \left(3a^3 + 5a - b + 2\right) \neq$ $a^3 - 5a + b - 2 - 3a^3 + 5a + b + 2 \neq$ $-2a^3 + 2b$
Rationale C	Did not distribute the negative sign correctly over all the terms in the second polynomial – applies to first three terms only. $\left(a^3 - 5a + b - 2\right) - \left(3a^3 + 5a - b + 2\right) \neq$ $a^3 - 5a + b - 2 - 3a^3 - 5a + b + 2 \neq$ $-2a^3 - 10a + 2b$

Instructional Implications: The key to success with this item relies on careful distribution of the negative. Careful addition of like terms completes the process.

Sample Solution:

$$\left(a^3 - 5a + b - 2\right) - \left(3a^3 + 5a - b + 2\right) =$$
$$a^3 - 5a + b - 2 - 3a^3 - 5a + b - 2 =$$
$$-2a^3 - 10a + 2b - 4$$

Item 47

Item Type: Multiple Choice

Calculator: Permitted

Benchmark: D1.b – Use measures of center and spread to compare and analyze data sets.

Correct Answer: C

Explanation of Distractors:

Rationale A	Uses the current average. $\dfrac{72+80+84+92}{4}=82$
Rationale B	Uses the desired average.
Rationale D	Gives an average score greater than or equal to 84 but it is not the minimum score necessary.

Instructional Implications: The underlying concept for this item is the computation of a mean as a ratio of the total of all scores to the number of scores. Students must then take care to correctly represent both of these sums and solve the resulting linear inequality.

Sample Solution: Adding the unknown score, *x*, increases the total number of scores, the divisor, to 5. This equation yields the minimum score of 92 for the last quiz.

$$\frac{72+80+84+92+x}{5}\geq 84$$
$$328+x\geq 84(5)$$
$$x\geq 92$$

American Diploma Project

ALGEBRA I

End-of-Course Exam

Practice Test Commentaries