This task was developed by high school and postsecondary mathematics and health sciences educators, and validated by content experts in the Common Core State Standards in mathematics and the National Career Clusters Knowledge & Skills Statements. It was developed with the purpose of demonstrating how the Common Core and CTE Knowledge & Skills Statements can be integrated into classroom learning – and to provide classroom teachers with a truly authentic task for either mathematics or CTE courses.

TASK: SPREAD OF DISEASE

TARGET COMMON CORE STATE STANDARD(S) IN MATHEMATICS:
N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*
A.SSE.3c Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
F.IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**
F.IF8b: Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*
F.BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
F.LE.4 For exponential models, express as a logarithm the solution to ab^{ct} =d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*
F.LE.5: Interpret the parameters in a linear or exponential function in terms of a context.*
TARGET STANDARDS FOR MATHEMATICAL PRACTICES:
MP 1. Make sense of problems and persevere in solving them.
MP.3 Construct viable arguments and critique the reasoning of others.
MP 4. Model with mathematics.
MP 5. Use appropriate tools strategically.
TARGET COMMON CORE STATE STANDARD(S) IN ELA/LITERACY:
RST.9-10.3 Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
WHST.9.10.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
WHST.11-12.7 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
TARGET CAREER AND TECHNICAL EDUCATION (CTE) KNOWLEDGE & SKILLS STATEMENTS:
HLC.01.01.02 Use knowledge of diseases and disorders to conduct health care role.
HLC.03 Solve problems using critical thinking skills (analyze, synthesize, and evaluate) independently and in teams. Solve problems using creativity and innovation.
HLC.06.01.01 Explain infection control practices and procedures.
HLC.06.01.06 Describe healthy behaviors.

RECOMMENDED COURSE(S):
Algebra II; Health Sciences unit on infectious diseases
ADDITIONAL INSTRUCTIONS:
This task should be completed in a class period with extra time allowed for the research and design needed to develop a flyer/poster/pamphlet.

* Modeling standards appear throughout the CCSS high school standards and are indicated by a star symbol (*).

About the Common Core State Standards in Mathematics
The Common Core State Standards (CCSS) for Mathematics are organized by grade level in grades K–8. At the high school level, the standards are organized by conceptual category (number and quantity, algebra, functions, geometry, and probability and statistics), showing the body of knowledge students should learn in each category to be college and career ready, and to be prepared to study more advanced mathematics. The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. www.corestandards.org

About the Common Core State Standards in English Language Arts/Literacy
The Common Core State Standards (CCSS) for ELA/Literacy are organized by grade level in grades K–8. At the high school level, the standards are organized by 9-10 and 11-12 grade bands. Across K-12 there are four major strands: Reading, Writing, Speaking and Listening, and Language. The CCSS also include Standards for Literacy in History/Social Studies, Science, and Technical Subjects, with content-specific (Reading and Writing) literacy standards provided for grades 6-8, 9-10, and 11-12, to demonstrate that literacy needs to be taught and nurtured across all subjects. www.corestandards.org

About the Career Cluster Knowledge and Skill Statements
As an organizing tool for curriculum design and instruction, Career Clusters™ provide the essential knowledge and skills for the 16 Career Clusters™ and their Career Pathways. It also functions as a useful guide in developing programs of study bridging secondary and postsecondary curriculum and for creating individual student plans of study for a complete range of career options. As such, it helps students discover their interests and their passions, and empowers them to choose the educational pathway that can lead to success in high school, college and career. http://www.careertech.org/career-clusters/resources/clusters/health.html. Although not included in this template, all Clusters and Pathways have Foundational Academic Expectations and Essential Knowledge & Skills Statements, which, in some cases, overlap with the Common Core State Standards.

KEY TERMS

- Direct contact disease
- Universal precautions
- Exponential equation

SPREAD OF DISEASE[1,2] – *The Task*

Disease can spread quickly without use of universal precautions. Suppose the spread of a direct contact disease in a stadium is modeled by the exponential equation $P(t) = 10{,}000/(1 + e^{3-t})$ where $P(t)$ is the total number of people infected after *t* hours. (Use the estimate for *e* (2.718) or the graphing calculator for *e* in your calculations.)

1. Estimate the initial number of people infected with the disease. Show how you found your answer.

2. Assuming the disease does not present symptoms for 24 hours, how many people will have been infected after 3 hours? Show how you found your answer.

3. What is the maximum number of people who can become infected? (Note: $e^{(3-t)}$ will approach 0 for very large values of *t*).

4. Explain why your answer for Question #3 is the maximum.

5. The stadium needs to warn its guests about a rapid disease spread if it affects over 800 people. After how many minutes should the stadium inform its guests of the disease? Show how you found your answer.

6. Create a flyer/poster/pamphlet describing the chain(s) of infection for a typical contact disease, the mode(s) of prevention, and what your school can do to limit the spread of disease/pathogens. Research will be required to verify flyer information and statistics. (Materials could be shared/posted throughout school)

[1] Adapted from http://yale.edu/ynhti/curriculum/units/2009/5/09.05.08.x.html
[2] A teacher could plug in any disease/infection and location above to make relevant to their coursework

SPREAD OF DISEASE – Possible Solutions

1. *Estimate the initial number of people infected with the disease. Show how you found your answer.*
   To solve for the initial number of people infected with the disease, the student must substitute in 0 for time (t).

   $P(0) = 10{,}000/(1 + e^{3}) \approx 474.26$
   About 474 people would be initially infected.

2. *Assuming the disease does not present symptoms for 24 hours, how many people will have been infected after 3 hours? Show how you found your answer.*
   Substitute 3 hours for t (time).

   $P(3) = 10{,}000/(1 + e^{0}) = 10{,}000/(1+1) = 10{,}000 / 2 = 5{,}000$
   We would expect about 5,000 people to be infected after 3 hours.

3. *What is the maximum number of people who can become infected? (*Note: $e^{(3-t)}$ will approach 0 for very large values of *t).*

   There are two ways to solve this question. We can either graph the equation to see that it approaches 10,000, or analyze the function algebraically:

   The end behavior function states that *e* to a negative number approaches 0. As *t* increases, the exponent becomes increasingly negative, which means that as *t* approaches infinity, the exponent becomes a smaller and smaller negative number, making the power expression approach 0 [as $t \to \infty$, $e^{(3-t)} \to 0$]. Therefore one can assume:

   $P(t \to \infty) = 10{,}000/(1 + 0) = 10{,}000$
   Therefore, the maximum number of people who can become infected is 10,000.

4. *Explain why your answer for Question #3 is the maximum.*

   Answers will vary.

   Students might include logic that involves the end behavior of the function as described in the solution for Question 3 or some discussion about the function itself and how the quotient could never be more than 10,000 since that number is divided by a value that must be positive and has an upper limit of 1.

   Students should recognize as part of this discussion that it appears that the number of the people in the stadium is 10,000, which would explain it as the maximum number of people who can become infected.

5. *The stadium needs to warn its guests about a rapid disease spread if it affects over 800 people. After how many minutes will the stadium need to inform its guests of the disease? Show how you found your answer.*

   Substitute 800 for P(t) in order to solve for time (t):

$800 = 10{,}000/(1 + e^{3-t})$

$800(1 + e^{3-t}) = 10{,}000$

$(1 + e^{3-t}) = 10{,}000 / 800$

$(1 + e^{3-t}) = 100/8 = 12.5$

$e^{3-t} = 12.5 – 1 = 11.5$

$\ln(e^{3-t}) = \ln(11.5)$

$3 – t = \ln(11.5)$

$-t = \ln(11.5) – 3$

$t = 3 – \ln(11.5)$

$t \approx 0.5576\ldots \approx 0.56$ hours   0.56 hours (60 min/hour) ≈ 33.46 minutes

The stadium needs to warn of the spreading disease in 0.56 hours or about 33 minutes.

6. Create a flyer/poster/pamphlet describing the chain(s) of infection for a typical contact disease, the mode(s) of prevention, and what your school can do to limit the spread of disease/pathogens.

Research will be required to verify flyer information and statistics. (Materials could be shared/posted throughout school)

SPREAD OF DISEASE - *Possible Extensions*

The extensions below represent potential ways in which mathematics and/or CTE teachers can build on the task above. All of the extensions are optional and can be used in the classroom, as homework assignments, and/or as long-term interdisciplinary projects.

1. Could this exponential equation $P(t) = 10{,}000/(1 + e^{3-t})$ be used to determine how long it would take for the entire world to become infected? Explain why or why not.

2. Research and write a letter to the stadium owner explaining possible legal ramifications that may be incurred by the stadium due to disease spread.

SPREAD OF DISEASE – *Appendix: Alignment Ratings*

The rating system used in the following charts is as follows:

3 EXCELLENT ALIGNMENT:
The content/performance of the task is clearly consistent with the content/performance of the Common Core State Standard.

2 GOOD ALIGNMENT:
The task is consistent with important elements of the content/performance of the CCSS statement, but part of the CCSS is not addressed.

1 WEAK ALIGNMENT:
There is a partial alignment between the task and the CCSS, however important elements of the CCSS are not addressed in the task.

N/A:
For Mathematical Practices a content rating does not apply.

In the charts **C = Content Rating** and **P = Performance Rating**

COLOR KEY

- **Black** = Part of CCSS/K&S Statement aligned to task
- **Gray** = Part of CCSS/K&S Statement ***not*** aligned to task

Task-to-Mathematical Practice Alignment Recording Sheet

Task Name	Aligned CCSS Mathematical Practice Standards	C	P	Alignment Comments (Standards selection, partial alignments, reasons for rating, etc.)	Task Comments (Strengths, weaknesses, possible improvements, effectiveness, etc.)
SPREAD OF DISEASE	**MP 1.** Make sense of problems and persevere in solving them.	N/A	3	Students must analyze the given information, the constraints, relationships, and goals of the task. They must make conjectures about the form and meaning of the solution and plan a solution pathway. They must check the reasonableness of their solution, continually asking themselves, "Does this make sense?" While the task requires routine math, the student must persevere in their calculations and reasoning.	This is a multi-stage problem with real life applications and considerations. Students must identify quantities using practical situations, apply an algebraic model, and perform accurate quantitative calculations. They must show their work and explain their reasoning at various steps in the process.
	MP.3 Construct viable arguments and critique the reasoning of others.	N/A	2	This task asks students to explain and show work but also, in Question 4, to justify their response to Question 3. There is no requirement to critique the reasoning of others.	
	MP 4. Model with mathematics.	N/A	3	For this task students must solve problems in everyday life, society and the workplace, be able to identify important quantities in a practical situation and map their relationships, and interpret mathematical results in the context of the situation and determine whether the results make sense. Students are provided with the equation to model the situation.	
	MP 5. Use appropriate tools strategically.	N/A	3	For this task student are likely to use a calculator to apply *e* and may use graphic methods to solve. Various tools might also be used in creating the flyer, poster, or pamphlet for Question 6.	

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Task-to-Common Core State Standards Alignment Recording Sheet

Task Name	Aligned CCSS Content Standards	C	P	Alignment Comments (Standards selection, partial alignments, reasons for rating, etc.)	Task Comments (Strengths, weaknesses, possible improvements, effectiveness, etc.)
SPREAD OF DISEASE	**N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*	3	3	As students substitute values into the exponential function, they must recognize the need to round to the nearest whole person. For Questions #3 and #4 students must think about the limits as time grows.	This complex real-world task combines calculation, algebraic modeling and analysis with work shown and explanations to support results.
	A.SSE.3c Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^{t} *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	3	3	For this task students must transform the exponential equations in order to solve the equation.	
	F.IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	3	3	Students are required to evaluate an exponential function at various values of its domain and to interpret the function to determine what happens for large values of *t*.	
	F.IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**	2	3	Students must relate the domain of the function to the quantities they are representing. They substitute domain values for Questions 1 and 2 and determine and explain the domain value that represents the maximum value of the function in Questions 3 and 4. They need to recognize and interpret the domain as units of time. There is no requirement to use a graph, even though students may do so.	
	F.IF8b: Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*	3	3	In Questions 3 and 4 of this task, students are asked to interpret the function as time increases. They must use the properties of exponents in their reasoning.	

Task Name	Aligned CCSS Content Standards	C	P	Alignment Comments (Standards selection, partial alignments, reasons for rating, etc.)	Task Comments (Strengths, weaknesses, possible improvements, effectiveness, etc.)
	F.BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	3	3	For this task, knowledge of the relationship between exponents and logarithms is essential to solving the exponential equation in Question 5.	
	F.LE.4 For exponential models, express as a logarithm the solution to ab^{ct} =d where a, c, and d are numbers and the base b is2, 10, or e; evaluate the logarithm using technology.*	3	3	In finding the solution for Question 5 of this task, students must express the equation using a natural log (ln) and would need to evaluate the ln expression using technology.	
	F.LE.5: Interpret the parameters in a linear or exponential function in terms of a context.*	3	3	In Question 3 the student finds that the maximum value for the function is one of the parameters. Then in Question 4 they must explain their answer, requiring an interpretation of that parameter in terms of the context.	

* Modeling standards appear throughout the CCSS high school standards and are indicated by a star symbol (*).

Task-to-National Career Cluster™ Knowledge & Skills Statements Alignment Recording Sheet

Task Name	Aligned National Career Cluster Knowledge & Skills Statements	C	P	Alignment Comments	Task Comments
SPREAD OF DISEASE	**HLC.01.01.02** Use knowledge of diseases and disorders to conduct health care role.	1	1	One of the sample indicators measures how students analyze methods to control the spread of pathogenic microorganisms and supports an aspect of question 6.	Extensions provided, or others created, based on this task could provide a deeper connection to the application of multiple aspects of Health Science and other Career Cluster™ areas. These include support services, emergency services, public health policy, and biotechnology.
	HLC.03 Solve problems using critical thinking skills (analyze, synthesize, and evaluate) independently and in teams. Solve problems using creativity and innovation.	3	3	The role of problem solving to determine the solution in the scenario can be done independently and in teams. Question 6 provides an opportunity to be creative and innovative in communicating the information on the spread of disease.	
	HLC.06.01.01 Explain infection control practices and procedures.	1	1	The task includes a component to help students understand the importance of infection control practices and procedures.	
	HLC.06.01.06 Describe healthy behaviors.	2	2	Question 6 builds out the task of researching (and communicating) information that encourages healthy behaviors in promoting disease prevention.	