Achieve ADP Algebra I
End-of-Course Exam Content Standards
with Comments & Examples

January 2010

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Change Log

In order to help track where changes and additions have been made to wording and examples throughout the document, this change log has been included.

Section	Column	Description	Date
Change Log	N/A	Updated.	January 2010
Table of Contents	N/A	Updated.	January 2010
Benchmark O2d	Right	Example and solution added.	January 2010
Benchmark L2c	Right	Example added.	January 2010
Benchmark D1c	Right	Bullet added to clarify the term “media” in the benchmark: “Media includes any report or data display that might be used in any published format, professional or student newspaper, student report at school, etc.”	January 2010
Benchmark D2a	Right	Example clarified: “Compare the number of ways the letters of the words FROG and DEER can be arranged to form unique *four-letter* password configurations.”	January 2010
Expectations of Knowledge	Algebra I Knowledge/Topics	Added: Order of operations. Bullet clarified: The squares of the integers 1 through 25, i.e., $1^2 = 1$, $2^2 = 4$, …, $25^2 = 625$.	January 2010
Expectations of Knowledge	Prior knowledge/topic	Added: volume and surface area of cylinders	January 2010

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About this document:

This version of the Algebra I End-of-Course Exam Content Standards includes two columns: The first column contains the standards, objectives and benchmarks; the second column includes explanatory comments, examples and limitations. The comments and examples are meant to add clarity to the meaning of the benchmarks for teachers and test item writers. Examples are provided only when necessary for clarity and are not meant to be exhaustive or to be used as sample test items. In some instances, in the standards and in the explanations, the word "including" is used followed by a list. The word "including" does not translate to "all inclusive" but rather means "including but not limited to." Some of the benchmarks have "assessment limitations" which means that the stated content is not tested on the ADP Algebra I End-of-Course Exam. However, this does not imply that teachers should not teach or expand on this content.

Background: The American Diploma Project (ADP) Network includes states dedicated to making sure every high school graduate is prepared for college and a career. In each state, governors, state superintendents of education, business executives, and college and university leaders are working to restore value to the high school diploma by raising the rigor of high school standards, assessments and curriculum, and better aligning these expectations with the demands of postsecondary education and careers.

In May 2005, leaders from several of the ADP Network states began to explore the possibility of working together, with support from Achieve, to develop a common end-of-course exam at the Algebra II level. These states were planning to require or strongly encourage students to take an Algebra II level course in order to better prepare them for college and careers, as Algebra II or its equivalent serves as a gateway course for higher education and teaches quantitative reasoning skills important for the workplace. State leaders recognized that using a common end-of-course test would help ensure a consistent level of content and rigor in classes within and across their respective states. They also understood the value of working collaboratively on a common test: the potential to create a high quality test faster and at lower cost to each state and to compare their performance and progress with one another. The development of the Algebra I end-of-course exam was a natural extension of this effort and was designed to support the goals of the Algebra II initiative. Leadership for the design of the content and format was provided by a subset of the state content leaders involved in the development of the Algebra II exam.

As an extension of the ADP Algebra II End-of-Course Exam, the ADP Algebra I End-of-Course Exam serves similar, parallel purposes:

1. ***To improve curriculum and instruction****—and ensure consistency within and across states***.** The exam will help classroom teachers focus on the most important concepts and skills in an Algebra I, or equivalent, class and identify areas where the curriculum needs to be strengthened. For schools administering both exams, the Algebra I Exam will compliment the Algebra II Exam and will help ensure a compatible, consistent and well-aligned Algebra curriculum. Once standards are set teachers will get test results back within three weeks of when the exam is administered, which will provide sufficient time to make the necessary adjustments for the next year's course.

2. ***To help high schools determine if students are ready for rigorous higher level mathematics courses.*** Because the test is aligned with the ADP mathematics benchmarks, it will measure skills students need to succeed in mathematics courses beyond Algebra I. High schools will be able to use the results of the exam to tell Algebra I students, parents, teachers and counselors whether a student is ready for higher level mathematics, or if they have content and skill gaps that need to be filled before they enroll in the next mathematics class in their high school's course sequence. This information should help high schools better prepare their students for upper level mathematics, which might include passing high school exit exams or state mathematics graduation exams. This will reduce the need for multiple retakes of courses or exams needed to graduate, hopefully avoiding remedial courses designed to review Algebra I skills and concepts.

3. ***To compare performance and progress among the participating states.*** Having agreed on the content expectations for courses at the Algebra I level, states are interested in tracking student performance over time. Achieve will issue a report each year comparing performance and progress among the participating states. This report will help state education leaders, educators and the public assess performance, identify areas for improvement and evaluate the impact of state strategies for improving secondary math achievement.

The Algebra I End-of-Course Exam will consist of a Algebra I skills and concepts that are typically taught in an Algebra I course, which will be taken by students across participating states. States not part of the development group may also decide to purchase and administer this test. The exam may be administered at any point in a student's course of studies from middle school through high school Algebra I or its equivalent.

The Algebra I End-of-Course Exam: The Algebra I End-of-Course Exam covers a range of algebraic topics. Successful students will demonstrate conceptual understanding of the properties and operations of real numbers with emphasis on ratio, rates, and proportion and numerical expressions containing exponents and radicals. They will be able to operate with polynomial expressions, factor polynomial expressions and use algebraic radical expressions. They will analyze, represent and graph linear functions including those involving absolute value and recognize and use linear models. They will solve and graph linear equations and inequalities and will be able to use them to represent contextual situations. They will solve systems of linear equations and model with single variable linear equations, one- or two-variable inequalities, or systems of equations. Successful students also will be able to demonstrate facility with estimating and verifying solutions of linear equations, making use of technology where appropriate to do so. Students will represent simple quadratic functions in multiple ways and use quadratic models, as well as solve quadratic equations. Finally, connections to algebra will be made through the interpretation of linear trends in data, the comparison of data using summary statistics, probability and counting principles, and the evaluation of data-based reports in the media.

There are a variety of types of test items that will assess this content, including some that cut across the objectives in a standard and require students to make connections and, where appropriate, solve rich contextual problems. The Algebra I End-of-Course Exam will include three types of items: multiple-choice items (worth 1 point each), short-answer items (worth 2 points each) and extended-response items (worth 4 points each). Approximately thirty percent of the student's score will be based on the short-answer and extended-response items. Although the test is untimed, it is designed to take approximately 120 minutes, comprised of two 60 minute sessions, one of which will allow calculator use. However, some students may require – and should be allowed – additional time to complete the test. Test items, in particular extended-response items, may address more than one content objective and benchmark within a standard. Each standard within the exam is assigned a priority, indicating the approximate percentage of points allocated to that standard on the test.

Algebra I End-of-Course Exam calculator policy: The appropriate and effective use of technology is an essential practice in the Algebra I classroom. At the same time, students should learn to work mathematically without the use of technology. Computing mentally or with paper and pencil is required on the Algebra I End-of-Course Exam and should be expected in classrooms where students are working at the Algebra I level. It is therefore important that the Algebra I End-of-Course Exam reflect both practices. For purposes of the Algebra I End-of-Course Exam, students are expected to have access to a calculator for one of the two testing sessions and the use of a graphing calculator is strongly recommended. Scientific or four-function calculators are permitted but not recommended because they do not have graphing capabilities. Students should not use a calculator that is new or different for them on the exam but rather should use the calculator they are accustomed to and use every day in their classroom work. For more information about technology use on the Algebra I End-of-Course Exam, see the ADP Algebra End-of-Course Exams Calculator Policy at www.achieve.org/AssessmentCalcPolicy.

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It will be necessary to clear the calculator memory, including any stored programs and applications, on all calculators both before and after the exam. Please be advised that the clearing of the calculator memory may permanently delete stored programs or applications. Students should be told prior to the test day to store all data and software they wish to save on a computer or a calculator not being used for the test. In some states, an IEP or 504 Plan may specify a student's calculator use on this Exam. Please check with your state's Department of Education for specific policies or laws.

Algebra I level curriculum: Modeling and problem solving are at the heart of the curriculum at the Algebra I level. Mathematical modeling consists of recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution and interpreting the solution in the context of the original problem. Students must be able to solve practical problems, representing and analyzing the situation using symbols, graphs, tables or diagrams. They must effectively distinguish relevant from irrelevant information, identify missing information, acquire needed information and decide whether an exact or approximate answer is called for, with attention paid to the appropriate level of precision. After solving a problem and interpreting the solution in terms of the context of the problem, they must check the reasonableness of the results and devise independent ways of verifying the results.

The standards included in this document are intended to reflect this curricular focus and to guide the work of the test designers and the test item developers. It is also the case that curriculum at the Algebra I level will include content and processes not included on the Algebra I End-of-Course Exam, as some are not easily assessed by a test of this nature. Problems that require extended time for solution should be addressed in the Algebra I level classroom, even though they cannot be included in this end-of-course exam.

Algebra I level classroom practices: Effective communication using the language of mathematics is essential in a class engaged in Algebra I level content. Correct use of mathematical definitions, notation, terminology, syntax and logic should be required in all work at the Algebra I level. Students should be able to translate among and use multiple representations of functions fluidly and fluently. They should be able to report and justify their work and results effectively. To the degree possible, these elements of effective classroom practice are reflected in the Algebra I End-of-Course Exam content standards.

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Algebra I End-of-Course Test Standards

O: Operations on Numbers and Expressions
O1. Number Sense and Operations
O1.a Reasoning with real numbers
O1.b Using ratios, rates, and proportions
O1.c Using numerical exponential expressions
O1.d Using numerical radical expressions

O2. Algebraic Expressions
O2.a Using algebraic exponential expressions
O2.b Operating with polynomial expressions
O2.c Factoring polynomial expressions
O2.d Using algebraic radical expressions

L: Linear Relationships
L1. Linear Functions
L1.a Representing linear functions in multiple ways
L1.b Analyzing linear function
L1.c Graphing linear functions involving absolute value
L1.d Using linear models

L2. Linear Equations and Inequalities
L2.a Solving linear equations and inequalities
L2.b Solving equations involving absolute value
L2.c Graphing linear inequalities
L2.d Solving systems of linear equations
L2.e Modeling with single variable linear equations, one- or two-variable inequalities or systems of equations

N: Non-linear Relationships
N1. Non-linear Functions
N1.a Representing quadratic functions in multiple ways
N1.b Distinguishing between function types
N1.c Using quadratic models

N2. Non-linear Equations
N2.a Solving literal equations
N2.b Solving quadratic equations

D: Data, Statistics and Probability
D1. Data and Statistical Analysis
D1.a Interpreting linear trends in data
D1.b Comparing data using summary statistics
D1.c Evaluating data-based reports in the media

D2. Probability
D2.a Using counting principles
D2.b Determining probability

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O: Operations on Numbers and Expressions Priority: 25% Successful students will be able to perform operations with real numbers, including numerical expressions involving exponents, scientific notation and square roots, using estimation and an appropriate level of precision. Reasoning skills will be emphasized, including justification of results. There is a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and, where appropriate, solve contextual problems.	
Content Benchmarks	**Explanatory Comments and Examples**
O1. Number Sense and Operations	
a. Use properties of number systems within the set of real numbers to verify or refute conjectures or justify reasoning and to classify, order, and compare real numbers.	• Define, give examples of, distinguish between and use numbers and their properties, from each of the following number sets: whole numbers, integers, rationals, irrationals, and reals. • Determine whether the square roots of whole numbers are rational or irrational. *Example*: Which of the following numbers are rational and which are irrational? Explain. $\sqrt{(10)(40)}, \quad \sqrt{4^3}, \quad 2\sqrt{2}, \quad \sqrt{49}$ • Compare and order real numbers, including determining between which two consecutive whole numbers the value of a square root lies. *Example*: Which of the following numbers comes closest to the value of π without exceeding it: $\sqrt{10}, \quad 3.14, \quad \text{and} \quad \frac{22}{7}$ • Provide counterexamples to refute a false conjecture. • Establish simple facts about rational and irrational numbers using logical arguments and examples. *Example*: Give an example to illustrate that if r and s are rational, then both $r + s$ and $(r)(s)$ are rational. *Sample Solution:* Both $\frac{3}{4}$ and 2.3 are rational; $\frac{3}{4} + 2.3 = \frac{3}{4} + \frac{23}{10} = \frac{15}{20} + \frac{46}{20} = \frac{61}{20}$ which is the ratio of two integers, hence rational.

	Assessment Limitation: Items involving radicals will be limited to square roots. Students will not be expected to produce formal proofs.
b. Use rates, ratios and proportions to solve problems, including measurement problems.	• Use dimensional analysis for unit conversion. • Solve problems using derived measures (Derived measures are those achieved through calculations with measurement that can be taken directly, e.g. percent change and density). • Solve problems involving scale factor (e.g., similar figures, scale drawings, map scales, dilations). • Solve applications related to proportional representation. *Example*: There are 223 students in the freshman class, 168 in the sophomore class, 173 in the junior class and 138 in the senior class. The student council has 30 members, with these seats allocated based on the number of students in each class. How many student council members should each class have? Explain your answer. • For applications, this includes using and interpreting appropriate units of measurement, estimation and the appropriate level of precision.
c. Apply the laws of exponents to numerical expressions with integral exponents to rewrite them in different but equivalent forms or to solve problems.	*Example:* Rewrite $\frac{7\left(3^{-3}\right)}{\left(2^{-4}\right)\left(3^{5}\right)}$ as a fraction having only positive exponents. *Sample Solution:* $\frac{7\cdot 3^{-3}}{2^{-4}\cdot 3^{5}}=\frac{7\cdot 2^{4}}{3^{3}\cdot 3^{5}}=\frac{7\cdot 2^{4}}{3^{8}}$ *Example:* Multiply, giving the answer without exponents: $\frac{2}{5^{-4}}\cdot\frac{3^{-3}}{5^{2}}\cdot\frac{3^{5}}{2^{5}}$ *Sample Solution:* $\frac{2}{5^{-4}}\cdot\frac{3^{-3}}{5^{2}}\cdot\frac{3^{5}}{2^{5}}=\frac{2\cdot 3^{5}\cdot 5^{4}}{3^{3}\cdot 5^{2}\cdot 2^{5}}=\frac{3^{2}\cdot 5^{2}}{2^{4}}=\frac{225}{16}$

	• Represent, compute and solve problems using numbers in scientific notation. Examples of applications may include determining national debt, astronomical distances or the distance between electrons and protons in an atom. • For applications, this includes using and interpreting appropriate units of measurement, estimation and the appropriate level of precision.
d. Use the properties of radicals to rewrite numerical expressions containing square roots in different but equivalent forms or to solve problems.	• Add, subtract, multiply, divide and manipulate numerical expressions with square roots. Results may be required to be given in exact form. *Example*: Show or explain how $\left(2\sqrt{2}\right)^2$ is equal to 8. *Sample Solution*: $\left(2\sqrt{2}\right)^2 = 4(2) = 8$ *Example:* Show or explain how $5\sqrt{2}$ is equal to $\sqrt{50}$. *Sample Solution:* Showing that the number on the left equals that on the right: $5\sqrt{2} = \sqrt{25}\cdot\sqrt{2} = \sqrt{50}$ *Example*: Rewrite the radicals to determine the sum of $\sqrt{8}+\sqrt{18}.$ *Sample solution*: $\sqrt{8}+\sqrt{18} = \sqrt{4\cdot 2}+\sqrt{9\cdot 2} = 2\sqrt{2}+3\sqrt{2} = 5\sqrt{2}$ *Example*: $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ *Example*: $\frac{\sqrt{6}+\sqrt{9}}{\sqrt{3}} = \sqrt{2}+\sqrt{3}$ • Use the distance formula, based on the Pythagorean Theorem, to solve problems. *Example*: Determine the perimeter of a quadrilateral with vertices (1, 1), (-1, 2), (2, 4) and (4, 3). *Example*: If the legs of a right triangle measure $\sqrt{5}$ units and $\sqrt{7}$ units, determine the exact measure of the hypotenuse in simplest form.

	Sample Solution: $\left(\sqrt{5}\right)^2 + \left(\sqrt{7}\right)^2 = 5 + 7 = 12$ The length of the hypotenuse is $\sqrt{12} = 2\sqrt{3}$ units. • For applications, this includes using and interpreting appropriate units of measurement for solutions. *Assessment Limitation*: When division by a radical or rationalization of a denominator is required, the denominator will be a monomial.
O2. Algebraic Expressions	
a. Apply the laws of exponents to algebraic expressions with integral exponents to rewrite them in different but equivalent forms or to solve problems.	*Example*: Write the expression in simplest form: $\left(2a^2b^3\right)^5 = 32a^{10}b^{15}$ *Example:* Write the expression in simplest form: $\frac{3a^2 + 6ab}{3a} = a + 2b$ • Translate to expressions with only positive exponents. *Example*: Rewrite the expression with each variable appearing only once and with only positive exponents: $\frac{3x^{-2}y^3}{2x^{-5}y^{-3}} = \frac{3}{2}x^3y^6$ • Translate to expressions with variables appearing only in the numerator. *Example*: Rewrite the expression with variables only in the numerator: $\frac{3s^3}{2r^5} = \frac{3}{2}s^3r^{-5}$ • For applications, this includes using and interpreting appropriate units of measurement, estimation and the appropriate level of precision. *Assumption*: All algebraic expressions are defined.
b. Add, subtract and multiply polynomial expressions with or without a context.	*Example*: $3x^5(x-2) - 2x^4\left(x^2+2\right)$ *Sample Solution*: $3x^5(x-2) - 2x^4\left(x^2+2\right) = 3x^6 - 6x^5 - 2x^6 - 4x^4 = x^6 - 6x^5 - 4x^4$

	Example: Multiply: $(x+a)(x+b)$ *Sample Solution:* $(x+a)(x+b) = x^2 + ax + bx + ab = x^2 + (a+b)x + ab$ *Assessment Limitation:* Multiplication is limited to a monomial multiplied by a polynomial or a binomial multiplied by a binomial.
c. Factor simple polynomial expressions with or without a context.	*Example:* Factor completely $6u^5 - 15u^3$ *Sample Solution:* $6u^5 - 15u^3 = 3u^3(2u^2 - 5)$ *Example*: Factor completely $3x^3y + 21x^2y + 30xy$ *Sample Solution:* $3xy(x^2 + 7x + 10) = 3xy(x+2)(x+5)$ *Example*: Factor completely $6x^2 - x - 12$ *Sample Solution:* $(3x+4)(2x-3)$ *Assessment Limitation:* Factoring will be limited to factoring out common monomial factors, perfect-square trinomials, differences of squares and quadratics of the form $ax^2 + bx + c$ that factor over the set of integers. The factoring process may require more than one step.
d. Use the properties of radicals to convert algebraic expressions containing square roots into different but equivalent forms or to solve problems.	• Add, subtract, multiply, divide and manipulate algebraic expressions with square roots. Results may be required to be given in exact form. • When taking square roots of variable expressions, absolute values must be included when appropriate. *Example:* $\sqrt{x^3} = x\sqrt{x}$ because $\sqrt{x^3}$ was assumed real $\sqrt{x^2} = \lvert x\rvert$, $\sqrt{x^4} = x^2$, $\sqrt{x^6} = \lvert x^3\rvert$ or $\lvert x\rvert^3$, $\sqrt{x^8} = x^4$, $\sqrt{x^{10}} = \lvert x^5\rvert$ or $\lvert x\rvert^5$ *Example*: Explain how $\sqrt{25x^6}$ is equal to $5\lvert x^3\rvert$ and to $5\lvert x\rvert^3$

	Sample Solution: $\sqrt{25x^6} = \sqrt{5^2 \cdot x^2 \cdot x^2 \cdot x^2}$ $= 5\lvert x \cdot x \cdot x \rvert = 5\lvert x^3 \rvert = 5\lvert x \rvert\lvert x \rvert\lvert x \rvert = 5\lvert x \rvert^3$ *Example:* Simplify completely $\sqrt{x^4 y^{-7}} \cdot \sqrt{x^{-6} y^5}$ *Solution*: $\sqrt{x^4 y^{-7}} \cdot \sqrt{x^{-6} y^5} = \sqrt{x^{-2} y^{-2}} = \sqrt{\dfrac{1}{x^2 y^2}} = \dfrac{1}{\lvert x \rvert y}$ • For applications, this includes using and interpreting appropriate units of measurement and the appropriate level of precision. *Assumption*: All radical expressions represent real numbers. *Assessment Limitation:* Expressions under radicals will be limited to monomials. When rationalization of a denominator is required, the radical in the denominator will contain no variables.

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L: Linear Relationships
Priority: 35%

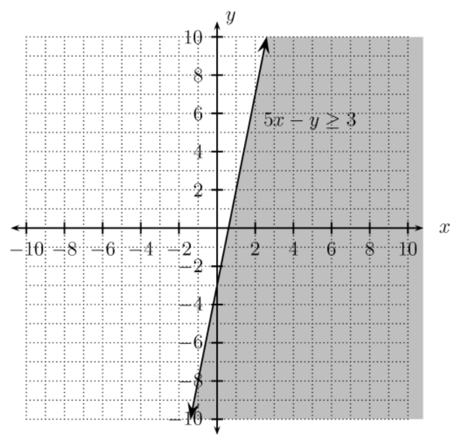
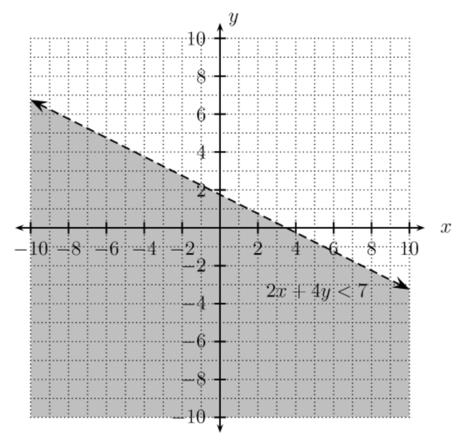
Successful students will be able to solve and graph the solution sets of linear equations, inequalities and systems of linear equations and to use words, tables, graphs and symbols to represent, analyze and model with linear functions. There is a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and, where appropriate, solve contextual problems. In contextual problems, students may be required to graph and interpret their solutions in terms of the context. They should be able to apply such problem solving heuristics as: identifying missing or irrelevant information; testing ideas; considering analogous or special cases; making appropriate estimates; using inductive or deductive reasoning; analyzing situations using symbols, tables, graphs or diagrams; evaluating progress regularly; checking for reasonableness of results; using technology appropriately; deriving independent methods to verify results; and using the symbols and terms of mathematics correctly and precisely. On the Algebra I End-of-Course test, function notation may be used.

Content Benchmarks	Explanatory Comments and Examples
L1. Linear Functions	
a. Recognize, describe and represent linear relationships using words, tables, numerical patterns, graphs and equations. Translate among these representations.	• Use correct terminology and notation for functions (e.g., $f(x)$, independent and dependent variables, etc.). When equations are presented, any form of a linear equation can be used. *Example:* Explain how the relationship between length of the side of a square and its perimeter can be represented by a direct proportion. • Use models and algebraic formulas to represent and analyze linear patterns, including determining a formula for the general term of an arithmetic sequence and interpreting the constant difference as the slope of the line that represents the pattern. *Example:* Given the sequence: 5, 7, 9, 11, … If 5 is considered the first term when $x = 1$, what linear equation could generate this pattern? *Example:* Express the following sentence in equation form: two times the quantity of a number increased by eight is equivalent to five less than the same number. *Sample Solution:* $2(x+8) = x-5$ • For items where a student is required to graph the equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc). *Assessment Limitation*: Subscript notation will not be used or required for items involving sequences.

b. Describe, analyze and use key characteristics of linear functions and their graphs.	• Key characteristics include constant slope and x- and y-intercepts. • When equations are presented, any form of a linear equation can be used. • Interpret slopes of given lines to determine whether lines are parallel, perpendicular, intersecting or coincident. *Example*: Write an equation for a line parallel to the line through (1, -2) and (-3, 5). • Identify and distinguish among parameters and the independent and dependent variables in a linear relationship. • Describe the effects of varying the parameters m and b in linear functions of the form $f(x) = mx + b$ or $y = mx + b$. *Example*: Compare and contrast the positions of the graphs for the following three functions and explain how the positions are related to the equations: $f(x) = 5x$, $g(x) = 5x + 2$, and $h(x) = 5x - 2$. • Apply direct proportions, as a special linear relationship, and analyze their graphs in a context.
c. Graph the absolute value of a linear function and determine and analyze its key characteristics.	• Key characteristics include vertex, slope of each branch, intercepts, domain and range, maximum, minimum, transformations, and opening direction. *Example*: Graph each of the following absolute value equations and compare and contrast the graphs with the graph of $p(x) = \|x\|$: $q(x) = -\|x\|$, $r(x) = \|2x\|$, $s(x) = \|x + 2\|$, and $t(x) = \|x\| + 2$ • For items where a student is required to graph the equation or function, axes and scales should be labeled.

d. Recognize, express and solve problems that can be modeled using linear functions. Interpret their solutions in terms of the context of the problem.	• Interpret slope and y-intercept in the context of a problem. When equations are presented, any form of a linear equation can be used. *Example*: The linear function $40t = d$ can be used to describe the motion of a certain car, where t represents the time in hours and d represents distance traveled, in miles. What does the coefficient, 40, represent in the equation? Include units with the answer. • For items where a student is required to graph the equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc). • For applications, this includes using and interpreting appropriate units of measurement, estimation and the appropriate level of precision. *Assessment Limitation*: Absolute value items will not be used in context.
L2. Linear Equations and Inequalities	
a. Solve single-variable linear equations and inequalities with rational coefficients.	• Solve multi-step equations and inequalities. *Example*: Solve the equation $\frac{x}{2} - \frac{x+1}{3} = 2$. *Sample Solution*: $\frac{x}{2} - \frac{x+1}{3} = 2$ $6\left[\frac{x}{2} - \frac{x+1}{3}\right] = 6[2]$ $3x - 2(x+1) = 12$ $3x - 2x - 2 = 12$ $x = 14$ • Represent solution sets for inequalities symbolically as intervals or graphically on a number line.

	Example: Solve $3 - x < 5$ *Sample Solution:* $3 - x < 5$ $-x < 2$ $x > -2$ *Example*: Solve $-3 \le 5x + 4 \le 24$ • Linear equations may have no solution (empty set), an infinite number of solutions (identity) or a unique solution. *Example*: Determine and explain the solutions for each of the following three equations: $A)\ x + 0 = x + 2$ $B)\ x + 0 = x$ $C)\ x + 0 = 2x$ *Sample Solution*: $A)\ x + 0 = x + 2$ $0 = 2$ $\therefore$ no solution $B)\ x + 0 = x$ $0 = 0$ $\therefore$ all real numbers $C)\ x + 0 = 2x$ $x = 0$ *Assessment Limitation*: Limited to single variable, first degree for both equations and inequalities.
b. Solve equations involving the absolute value of a linear expression.	• Determine all possible values in the solution. *Example: Solve:* $\lvert x + 3 \rvert = 7$. *Sample Solutions*: $\lvert x + 3 \rvert = 7$ $x + 3 = 7$ or $x + 3 = -7$ $x = 4$ or -10 OR

	Since $\lvert x-b \rvert$ can be interpreted as the distance from x to b, the solutions of the above absolute value equation may be interpreted as the numbers, x, that are 7 units from –3. (i.e. $x=-3+7=4$ or $x=-3-7=-10$). *Assessment Limitation:* Equations will include only one absolute value expression and will be one of the following forms: $\lvert ax+b \rvert = c, \quad a\lvert x+b \rvert = c, \quad \lvert ax \rvert + b = c, \quad \lvert ax+b_1 \rvert + b_2 = c.$
c. Graph and analyze the graph of the solution set of a two-variable linear inequality.	• Represent algebraic solutions graphically on a coordinate plane. • For graphs of two-variable inequalities use shaded half-plane with solid or open boundary. *Example*: Graph $5x-y\geq 3$ *Solution*:  *Example*: Graph $2x+4y<7$ *Solution*:  • Provide examples of ordered pairs that are included in the solution set of a two-variable linear inequality. *Example*: Determine a point in the solution set for $3x+2y<6$.

	Example: Consider the inequality $6x + y < p$, where p is a constant. What must be true about the value of p in order for the origin to be part of the solution? • For items where a student is required to graph the equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).
d. Solve systems of linear equations in two variables using algebraic and graphic procedures.	• Systems of equations may include intersecting, parallel or coincident lines, some of which may be equations of horizontal or vertical lines. • For items where a student is required to graph the equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).
e. Recognize, express and solve problems that can be modeled using single-variable linear equations; one- or two-variable inequalities; or two-variable systems of linear equations. Interpret their solutions in terms of the context of the problem.	*Example*: Jim spent \$200 on gifts for his family. He spent the money on toys, clothes and a \$15 DVD. He spent 4 times as much on clothes as he did on toys. Write an equation in one variable that can be used to determine how much money Jim spent on toys. Solve the equation to determine how much Jim spent on toys. *Example*: A triangle is formed by the intersections of the x-axis, the y-axis and the line $2x + 3y = 6$. What is the area of the triangle? *Example*: One angle of an acute triangle is twice the first angle while the third angle is 40º more than the first angle. Determine the degree measure of each of the three angles. *Example:* Quick Trip rental car agency charges a flat weekly rate of \$193.00 and \$0.19 per mile. Drive Easy rental car agency charges a flat weekly rate of \$219.00 and \$0.15 per mile for an identical car. For a one-week rental, how many miles does the car need to be driven so that the charges for a rental at Quick Trip are the same as a rental at Drive Easy? • For items where a student is required to graph the equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc). • For applications, this includes using and interpreting appropriate units of measurement, estimation and the appropriate level of precision.

Achieve ADP Algebra I End-of-Course Exam Content Standards with Comments & Examples

January 2010

N: Non-linear Relationships Priority: 20%	
Successful students will be able to recognize, represent, analyze, graph, solve equations and apply non-linear functions, including quadratic and exponential. There is a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and, where appropriate, solve contextual problems. In contextual problems students will be required to graph and interpret their solutions in terms of the context. They should be able to apply such problem solving heuristics as: identifying missing or irrelevant information; testing ideas; considering analogous or special cases; making appropriate estimates; using inductive or deductive reasoning; analyzing situations using symbols, tables, graphs or diagrams; evaluating progress regularly; checking for reasonableness of results; using technology appropriately; deriving independent methods to verify results; and using the symbols and terms of mathematics correctly and precisely. On the Algebra I End-of-Course test function notation may be used.	
Content Benchmarks	**Explanatory Comments and Examples**
N1. Non-linear Functions (In this section, all coefficients will be integers.)	
a. Recognize, describe, represent and analyze a quadratic function using words, tables, graphs or equations.	• Use correct terminology and notation for functions (e.g. $f(x)$, independent and dependent variables, etc). • Determine and analyze key characteristics of quadratic functions and their graphs (e.g. axis of symmetry, vertex, zeros, y-intercept, domain, range, maximum, minimum, opening direction, etc.). • Sketch a graph of a quadratic equation using the zeros and vertex when given the equation. *Example*: Determine the vertex of the function $f(x)=4x^2-8x-5$ *Sample Solutions*: $f(x)=4x^2-8x-5$ $f(x)=(2x-5)(2x+1)$ $0=(2x-5)(2x+1)$ $x=\left\{-\frac{1}{2},\ \frac{5}{2}\right\}$

	To find the x-value of the vertex, average the zeros: $x = \frac{\left(-\frac{1}{2}+\frac{5}{2}\right)}{2} = 1$ $f(1) = 4(1)^2 - 8(1) - 5 = -9$ $(1,\ -9)$ OR Substitute 4 and -8 into $x = \frac{-b}{2a}$, and then solve for $f(x)$. $x = \frac{-(-8)}{2(4)} = 1$ $f(1) = 4(1)^2 - 8(1) - 5 = -9$ $(1,\ -9)$ • For items where a student is required to graph the equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc). *Assessment Limitations*: In constructed response items, students will not be required to derive quadratic equations from tables, graphs or words. Completing the square will not be required. Quadratic functions will have integral coefficients. When the vertex must be determined, the vertex of a quadratic function must have integral values. When zeros are to be determined or used, the zeros of the quadratic function must be rational. Quadratic functions may be represented in the following forms polynomial $\left(f(x) = ax^2 + bx + c\right)$ or factored $\left(f(x) = a(x-r)(x-s)\right)$.

b. Analyze a table, numerical pattern, graph, equation or context to determine whether a linear, quadratic or exponential relationship could be represented. Or, given the type of relationship, determine elements of the table, numerical pattern or graph.	• Distinguish between types of functions, including linear, quadratic, and exponential. • Recognize when an exponential model is appropriate (growth or decay). • Determine if an exponential function is increasing or decreasing. • Students may be required to explain their reasoning. *Example*: Given the following increasing numerical pattern, determine the type of relationship that exists (linear, quadratic or exponential) and justify your conclusion. 3, 6, 12, 24, 48, … • Extend a table, numerical pattern or graph given the type of relationship (quadratic or exponential). • Use first and second differences to determine the type of function represented. *Assessment Limitation*: Exponential functions in the form $y = ab^x$ will include rational non-zero values for both a and b, where $b > 0$. When exponents are specifically named for exponential functions, the exponents will be integers.
c. Recognize and solve problems that can be modeled using a quadratic function. Interpret the solution in terms of the context of the original problem.	• For physics applications, formulas will be provided (e.g., $s = -16t^2 + 48t + 64$). • For applications, this includes using and interpreting appropriate units of measurement, estimation and the appropriate level of precision. • For items where a student is required to graph the equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc). *Assessment Limitation*: Contexts will be accessible for students working at this level (e.g. area, Pythagorean relationships or motion). No formal physics notation will be used (e.g. v_0, s_0, etc). Quadratic functions will have integral coefficients. When the vertex must be determined, the vertex of the quadratic function must have integral values. When zeros are to be determined, or used, the zeros of the quadratic function must be rational.

N2. Non-linear Equations (In this section, all coefficients will be integers.)	
a. Solve equations involving several variables for one variable in terms of the others.	*Example*: Solve for *r:* $V = \pi r^2 h$ *Example*: Solve for *y:* $z = 3x^2 y + 4y$ *Assumption*: All algebraic functions are defined. All radical expressions represent real numbers. *Assessment Limitation*: Equations may contain variables to a power higher than the second degree, but students will not be asked to solve for any variable that is higher than the second degree.
b. Solve single-variable quadratic equations.	*Example*: Solve the following for x: $x(2x+5)=0$ *Example*: Solve the following for x: $3x^2 - x - 10 = -8$ *Assessment Limitation*: Quadratic equations will have integral coefficients and rational solutions. Students may use any valid method to determine solutions for a quadratic equation.

Achieve ADP Algebra I End-of-Course Exam Content Standards with Comments & Examples

January 2010

D: Data, Statistics and Probability

Priority: 20%

Successful students will be able to apply algebraic knowledge to the interpretation and analysis of data, statistics and probability. Analysis and interpretation of univariate and bivariate data includes the use of summary statistics for sets of data and estimation of lines of best fit. While some important components in the study of data and statistics, such as misleading uses of data, sampling techniques, bias, question formulation and experiment design are addressed when possible in this Algebra I End-of-Course Exam, those topics will be expected to be assessed in more depth in the classroom. These benchmarks are intended to support and reinforce algebra concepts. For this reason, several sample algebraic solutions are provided for examples. There is a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and, where appropriate, solve contextual problems. In contextual problems, students will be required to graph and interpret their solutions in terms of the context.

Content Benchmarks	Explanatory Comments and Examples
D1: Data and Statistical Analysis	
a. Interpret and compare linear models for data that exhibit a linear trend including contextual problems.	• Create scatter plots and estimate a line of best fit. • Describe the correlation of data. • Interpret the slope and y-intercept of the line of best fit (regression line) in the context of the model • Use lines of best fit to extrapolate or interpolate within the range of the data and within the context of the problem. Determine when, within the context of a problem, it may be unreasonable to extrapolate beyond a certain point. *Example*: If a linear trend describes population growth in a small town over 5 years, explain why it would not be best to use the same linear trend to predict population in the town after 100 years. • For items where a student is required to model data with a graph, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc). *Assessment Limitation*: Students will not be required to calculate the correlation coefficient. Students will not be required to use regression to calculate a line of best fit. In items, it may be helpful for students to sketch a line of best fit to interpret the behavior of the data, however, students will not be required to draw the line of best fit for credit.

b. Use measures of center and spread to compare and analyze data sets.	• Analyze data sets and use summary statistics to compare the data sets and to answer questions regarding the data. *Example*: A student has scores of 78, 82, 91, 84 and 67 on the first five tests in a semester. What score must she earn on the sixth test in order to raise her average to 82? *Sample Solution*: $\frac{78+82+91+84+67+x}{6}=82$ $\Rightarrow 402+x=6(82)$ $\Rightarrow x=492-402=90$ • Determine the effects of outliers on statistics. *Example*: Explain what happens to the mean, median and mode when the same value, x, is added to each data point. *Example*: Given the following data set: 55, 55, 57, 58, 60 and 63. Describe how the measures of center or spread will or will not change if an additional data point of 57.5 is included with the set. *Assessment Limitation*: No item will assess only the calculations of mean, median or mode. Items will require the use of these concepts and/or calculations and will be at an appropriate cognitive level and difficulty for Algebra I. Measures of spread are limited to range.
c. Evaluate the reliability of reports based on data published in the media.	• Explain the impact of bias and the phrasing of questions asked during data collection. • Identify and explain misleading uses of data and data displays. • Analyze the appropriateness of a data display and the reasonableness of conclusions based on statistical studies. • Explain the difference between randomized experiments and observational data. • Media includes any report or data display that might be used in any published format, professional or student newspaper, student report at school, etc.

D2: Probability	
a. Use counting principles to determine the number of ways an event can occur. Interpret and justify solutions.	• Use understanding of permutations and combinations to solve problems with and without replacement. *Example:* Compare the number of ways the letters of the words FROG and DEER can be arranged to form unique four-letter password configurations. Explain your answer. *Example*: If a person has twice as many shirts as pairs of pants, how many different combinations can be made of a shirt and pair of pants, based on the number of pants? Sample Solution: $p =$ number of pairs of pants $2p =$ number of shirts $p(2p) = 2p^2 =$ number of combinations of pants and shirts *Assumption:* All spinners, number cubes and coins are fair unless otherwise noted. *Assessment Limitation*: Neither factorial notation nor factorial forms of formulas for combination $\left({}_nC_r\right)$ or permutation $\left({}_nP_r\right)$ will be used in items or be required to solve items on the test, however, students may use any valid method to solve the problem. Numbers involved will be manageable without formulas.
b. Apply probability concepts to determine the likelihood an event will occur in practical situations.	• Determine, exactly or approximately, the probability that an event will occur based on simple experiments (e.g., tossing number cubes, flipping coins, spinning spinners), counting principles or data. *Example*: If there are four brown, four black and four blue socks in a drawer, what is the probability that a matched pair will be selected when drawing out first one and then another, without replacing the first sock or being able to see the socks as they are drawn? • Make predictions based on experimental and theoretical probabilities and compare results. *Example:* In a sample of 100 randomly selected students, 37 of them could identify the difference in two brands of soft drinks. Based on these data, what is the best estimate of how many of the 2352 students in the school could distinguish between the soft drinks?

	Sample Solution: $\frac{37}{100} = \frac{x}{2352}$ $100x = 37(2352)$ $x = 870.24$ $\therefore$ 870 students would be expected to distinguish between the soft drinks *Assumption*: All events are equally likely and samples are representative of the population, unless otherwise stated. All spinners, number cubes and coins are fair unless otherwise noted.

ADP Algebra I End-of-Course Exam
Expectations of Knowledge

The ADP Algebra I End-of-Course Exam will assess students across a variety of algebra topics and within various contexts. Unlike some state or classroom assessments, a formula sheet will **not** be provided for students to use on this exam. Therefore, the following topics and formulas are provided here to enable teachers to appropriately prepare students for what is expected of them on the exam.

Algebra I knowledge/topics:

- Substitution
- Order of operations
- Quadratic Formula (Quadratic equations may be solved in multiple ways, however if a student chooses to use this method, the formula will not be provided. For this assessment, solutions will be rational.)
- The squares of the integers 1 through 25, i.e., $1^2 = 1$, $2^2 = 4$, …, $25^2 = 625$
- Approximate square roots (which two consecutive whole numbers a square root lies between)
- Forms of a linear equation: standard, slope-intercept, point-slope
- Distance = rate x time
- Distance formula (distance between two points on a line)
- For items where a student is required to graph the equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc). Students are expected to graph the solution set over the set of real numbers to indicate the key characteristics of the graph, unless the domain is restricted by the content of the item.

Assumption for Test Items:

For purposes of this exam, the following assumptions are made about the test items, without being explicitly stated in each item. However, if an assumption is not to be made for a particular item, the item will state the parameters to be considered.

- All algebraic expressions are defined.
- All radical expressions represent real numbers.
- All graphs are graphed over the set of real numbers.
- All spinners, number cubes and coins are fair.
- All events are equally likely and samples are representative of the population.
- All selections from a box, bag, bowl, etc. are considered random selections, without looking.

Use of π:

When specified that an exact answer is required, answers should be expressed in terms of π. If not specified, answers may be expressed in terms of π, or 3.14 or 22/7 may be used as an approximation for π.

ADP Algebra I End-of-Course Exam
Expectations of Knowledge

Prior knowledge/topic:

The following topics are mathematical concepts with which students entering an Algebra I curriculum should be familiar from prior mathematics courses. The high school curriculum or course sequence that a student might follow that leads them to this exam varies by state, district, and sometimes even school. Regardless of the course sequence followed, the mathematical concepts below are typically considered middle school concepts and taught before the Algebra I, or its equivalent, course(s).

- Definition of polygons, through octagon
- Perimeter of polygons
- Area of parallelograms
- Area of trapezoids
- Area and circumference of circles
- Area of triangles (not requiring trigonometry)
- Volume of cylinders and rectangular prisms
- Surface area of cylinders and right prisms with rectangular or triangular bases
- Definitions of basic geometric figures: line, line segment, ray, parabola
- Pythagorean Theorem
- Similar figures
- Scale factors
- Sum of the interior angles of a triangle equals 180°
- Simple and compound interest

<u>Standard measurement conversions</u>

12 inches = 1 foot
3 feet = 1 yard
5,280 feet = 1 mile

8 ounces = 1 cup
2 cups = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

16 ounces = 1 pound
2,000 pounds = 1 ton

<u>Metric conversions</u>

Using liters, meters, and grams
10 milli = 1 centi
10 centi = 1 deci
10 deci = 1 base
10 base = 1 deca
10 deca = 1 hecto
10 hecto = 1 kilo

<u>Time</u>

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
For purposes of this test, assume 1 year to be 365 days, 52 weeks, or 12 months.

ADP Algebra I and Algebra II End-of-Course Exam Notation Information

The information below is meant to inform teachers, schools, districts, and states about the notation **that students will see on the ADP Algebra I and Algebra II End-of-Course Exams.** It is not meant to exclude acceptable notation from being used in the classroom, but to let teachers know what students should expect to see on the exam. We expect students to be exposed to and use multiple forms of correct notation in their classrooms. In addition, students may answer items using any acceptable form of notation on the exam.

Notation on Both Algebra I and Algebra II End-of-Course Exams

Absolute value functions: $f(x) = -3\left|x + 2\right| + 1$

Set notation: $\{-1, 0, 4\}$ for solution sets

Negative fractions and rational expressions: $-\frac{2}{3}$ $\quad -\frac{x+1}{x}$

Monomials involving roots and exponents: $x^2 y\sqrt[4]{z^3}$
(x squared multiplied by y multiplied by the fourth root of z cubed)

Notation on Algebra II End-of-Course Exams – Core

Piecewise functions: $f(x) = \begin{cases} x - 2, & x \le 0 \\ x + 2, & x > 0 \end{cases}$

Greatest Integer Function: $f(x) = [x]$

Exponential functions of base e: $f(x) = 2e^{-0.024x}$

Domain restrictions: When a student is asked only to simplify a rational expression, all expressions are assumed to be defined. If a student is identifying an equivalent expression or solving equations, restrictions on the domain will be a part of the item, either in the question or the answer or both.

Composition of functions: $f(g(x))$

Variables with subscripts involving exponents: $v_0{}^2$ $\quad R_1{}^2$

ADP Algebra I and Algebra II End-of-Course Exam Notation Information

Notation on Algebra II End-of-Course Exams – Modules

Correlation coefficient: r

Combinations/Permutations: ${}_5C_2$ ${}_5P_2$

Matrix elements: a_{32} for the element in row 3 column 2

Vectors: $(0, 1)$ $\mathbf{u}$ $\mathbf{AB}$ $\overrightarrow{\mathbf{AB}}$

Spreadsheet applications: A1 for column A row 1; * for multiplication; / for division; ^ for exponents